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ON EXPONENTIAL STABILITY OF VARIATIONAL NONAUTONOMOUS DIFFERENCE EQUATIONS IN BANACH SPACES*

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Abstract

In this paper we study two concepts of exponential stability for variational nonautonomous difference equations in Banach spaces. Characterizations of these concepts are given. The obtained results can be considered as generalizations for variational nonautonomous difference equations of some well-known theorems due to Barbashin and Datko .

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1 Introduction

We start with some notations. Let $\mathbf{N}$ be the set of all positive integer and let Δ respectively T be the sets defined by

$$\Delta = \left\{ (m,n) \in \mathbf{N}^2, \text{ with } m \geq n \right\}$$

respectively

$$T = \left\{ (m,n,p) \in \mathbf{N}^3, \text{ with } m \geq n \geq p \right\}.$$

Let (X,d) be a metric space and V a real or complex Banach space. The norm on V and on $\mathcal{B}(V)$ (the Banach algebra of all bounded linear operators on V) will be denoted by $\|\cdot\|$.

Definition 1 *A mapping $\varphi : \Delta \times X \to X$ is called a discrete evolution semiflow on X if the following conditions hold:*
s_1) $\varphi(n,n,x) = x$, for all $(n,x) \in \boldsymbol{N} \times X$;
s_2) $\varphi(m,n,\varphi(n,p,x)) = \varphi(m,p,x)$, for all $(m,n,p,x) \in T \times X$.

Example 1 *Let $f : \boldsymbol{R}_+ \to \boldsymbol{R}$ be a bounded function and for $s \in \boldsymbol{R}_+$ we denote $f_s(t) = f(t+s)$ for all $t \in \boldsymbol{R}_+$. Then $X = \{f_s,\ s \in \boldsymbol{R}_+\}$ is a metric space with the metric $d(x_1,x_2) = \sup\limits_{t \in \boldsymbol{R}_+} |x_1(t) - x_2(t)|$.*

The mapping $\varphi : \Delta \times X \to X$ defined by $\varphi(m,n,x) = x_{m-n}$ is a discrete evolution semiflow.

Given a sequence $(A_m)_{m \in \mathbf{N}}$ with $A_m : X \to \mathcal{B}(V)$ and a discrete evolution semiflow $\varphi : \Delta \times X \to X$, we consider the problem of existence of a sequence $(v_m)_{m \in \mathbf{N}}$ with $v_m : \mathbf{N} \times X \to X$ such that

$$v_{m+1}(n,x) = A_m(\varphi(m,n,x))v_m(n,x)$$

for all $(m,n,x) \in \Delta \times X$. We shall denote this problem with (A,φ) and we say that (A,φ) is a *variational (nonautonomous) discrete-time system.*

For $(m,n) \in \Delta$ we define the application $\Phi_m^n : X \to \mathcal{B}(V)$ by

$$\Phi_m^n(x)v = \begin{cases} A_{m-1}(\varphi(m-1,n,x)) \dots A_{n+1}(\varphi(n+1,n,x)) A_n(x)v, & \text{if } m > n \\ v, & \text{if } m = n. \end{cases}$$

Remark 1 *From the definitions of* v_m *and* Φ_m^n *it follows that:*
c_1) $\Phi_m^m(x)v = v$, *for all* $(m,x,v) \in \boldsymbol{N} \times X \times V$;
c_2) $\Phi_m^p(x) = \Phi_m^n\left(\varphi(n,p,x)\right)\Phi_n^p(x)$, *for all* $(m,n,p,x) \in T \times X$;
c_3) $v_m(n,x) = \Phi_m^n(x)v_n(n,x)$, *for all* $(m,n,x) \in \Delta \times X$.

Definition 2 *A mapping* $\Phi : \Delta \times X \to \mathcal{B}(V)$ *is called a discrete evolution cocycle over discrete evolution semiflow* $\varphi : \Delta \times X \to X$ *if the following properties hold:*
c_1) $\Phi(n,n,x) = I$ *(the identity operator on V), for all* $(n,x) \in \boldsymbol{N} \times X$
and
c_2) $\Phi(m,p,x) = \Phi(m,n,(\varphi(n,p,x))\Phi(n,p,x)$, *for all* $(m,n,p,x) \in T \times X$.
If Φ *is a discrete evolution cocycle over discrete evolution semiflow* φ, *then the pair* $S = (\Phi,\varphi)$ *is called a discrete skew-evolution semiflow on X.*

Remark 2 *From Remark 1 it results that the mapping*

$$\Phi : \Delta \times X \to \mathcal{B}(V),\ \Phi(m,n,x)v = \Phi_m^n(x)v$$

is a discrete evolution cocycle over discrete evolution semiflow φ.

The concept of evolution cocycle was introduced by Megan and Stoica in [4]. It generalizes the classical notion of linear skew-product semiflows and evolution operators.

There are two remarkable stability criteria regarding the uniform exponential stability of solutions to the linear differential equations $x' = A(t)x$ on the half line, due to Barbashin ([1]) and Datko ([3]).

In this work we consider the classical concept of uniform exponential stability and a concept of nonuniform exponential stability introduced by Barreira and Valls ([2]) for the general case of variational nonautonomous discrete-time systems in Banach spaces.

The main goal of the paper is to present discrete-time versions of the Barbashin's and Datko's theorems for these stability concepts.

Continuous time versions of these results were obtained by Megan and Stoica in [9] and [10].

We remark that our proofs are not discretizations of the proofs from [9] and [10].

Other results about uniform exponential stability of discrete evolution semiflows were obtained by Pham Viet Hai in [6], [7] and [8].

2 Uniform exponential stability

Let (A, φ) be a discrete variational system associated to the discrete evolution semiflow $\varphi : \Delta \times X \to X$ and to the sequence of mappings $A = (A_m)$, where $A_m : X \to \mathcal{B}(V)$, for all $m \in \mathbf{N}$.

Definition 3 *The system (A, φ) is said to be uniformly exponentially stable (and denote u.e.s.) if there are the constants $N \geq 1$ and $\alpha > 0$ such that:*

$$e^{\alpha(m-n)} \left\| \Phi_m^n(x)v \right\| \leq N \left\| v \right\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$.

Remark 3 *It is easy to see that (A, φ) is uniformly exponentially stable if and only if there are $N \geq 1$ and $\alpha > 0$ with*

$$e^{\alpha(m-n)} \left\| \Phi_m^p(x)v \right\| \leq N \left\| \Phi_n^p(x)v \right\|$$

for all $(m, n, p, x, v) \in T \times X \times V$.

Example 2 *Let $\mathcal{C} = \mathcal{C}(\boldsymbol{R}_+, \boldsymbol{R})$ be the metric space of all continuous functions $x : \boldsymbol{R}_+ \to \boldsymbol{R}$, with the topology of uniform convergence on compact subsets of $\boldsymbol{R}_+$. $\mathcal{C}$ is metrizable relative to the metric given in Example 1*

Let $f : \boldsymbol{R}_+ \to (0, \infty)$ be a decreasing function with the property that there exists $\lim\limits_{t\to\infty} f(t) = \alpha > 0$. We denote by X the closure in $\mathcal{C}$ of the set $\{f_t, t \in \boldsymbol{R}_+\}$, where $f_t(s) = f(t+s)$, for all $s \in \boldsymbol{R}_+$. The mapping $\varphi : \Delta \times X \to X$ defined by $\varphi(m, n, x) = x_{m-n}$ is a discrete evolution semiflow.

Let us consider the Banach space $V = \boldsymbol{R}$ and let $A : X \to \mathcal{B}(V)$ defined by

$$A(x)v = e^{-\int_0^1 x(\tau)d\tau} v$$

for all $(x, v) \in X \times V$.

Then we obtain

$$\Phi_m^n(x)v = \begin{cases} e^{-\int_0^{m-n} x(\tau)d\tau} v, & \text{if } m > n \\ v, & \text{if } m = n \end{cases}$$

for all $(m, n, x, v) \in \Delta \times X \times V$. *Because* $x(\tau) \geq \alpha$ *we have that*

$$|\Phi_m^n(x)v| \leq e^{-\alpha(m-n)} |v|$$

for all $(m, n, x, v) \in \Delta \times X \times V$, *and hence* (A, φ) *is u.e.s.*

A characterization of the uniform exponential stability property is given by

Lemma 1 *The system* (A, φ) *is uniformly exponentially stable if and only if there exists a decreasing sequence of real numbers* (a_n) *with* $a_n \to 0$ *such that:*

$$\|\Phi_m^p(x)v\| \leq a_{m-n} \|\Phi_n^p(x)v\|$$

for all $(m, n, p, x, v) \in T \times X \times V$.

Proof. *Necessity.* It is a simple verification for $a_n = Ne^{-\alpha n}$, where N and α are given by Definition 3.

Sufficiency. If $a_n \to 0$ then there exists $k \in \mathbf{N}^*$ with $a_k < 1$. Then, for every $(m, n) \in \Delta$ there exist $p \in \mathbf{N}$ and $r \in [0, k)$ such that $m = n + pk + r$. From hypothesis and Remark 1 we obtain

$$\|\Phi_m^n(x)v\| = \left\|\Phi_{n+pk+r}^{n+pk}(\varphi(n + pk, n, x)) \Phi_{n+pk}^n(x)v\right\| \leq$$

$$\leq a_r \left\|\Phi_{n+pk}^n(x)v\right\| \leq a_0 \left\|\Phi_{n+pk}^{n+(p-1)k}(\varphi(n + (p-1)k, n, x)) \Phi_{n+(p-1)k}^n(x)v\right\| \leq$$

$$\leq a_0 a_k \left\|\Phi_{n+(p-1)k}^n(x)v\right\| \leq \ldots \leq a_0 a_k^p \|v\| =$$

$$= a_0 a_k^{\frac{m-n-r}{k}} \|v\| \leq a_0 e^{\alpha k} e^{-\alpha(m-n)} \|v\| \leq Ne^{-\alpha(m-n)} \|v\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$, where $N = 1 + a_0 e^{\alpha k}$ and $\alpha = -\frac{\ln a_k}{k}$.

Theorem 1 *For every system* (A,φ) *the following assertions are equivalent:*
(i) (A,φ) *is uniformly exponentially stable;*
(ii) there exist $d>0$ *and* $D\geq 1$ *such that:*

$$\sum_{k=n}^{\infty} e^{d(k-n)}\left\|\Phi_k^n(x)v\right\|\leq D\left\|v\right\|$$

for all $(n,x,v)\in \boldsymbol{N}\times X\times V$;
(iii) there exists $D\geq 1$ *such that:*

$$\sum_{k=n}^{\infty}\left\|\Phi_k^n(x)v\right\|\leq D\left\|v\right\|$$

for all $(n,x,v)\in \boldsymbol{N}\times X\times V$.

Proof. $(i)\Rightarrow(ii)$ It is a simple verification for $d\in(0,\alpha)$ and $D=\frac{N}{1-e^{d-\alpha}}$, where N and α are given by Definition 3.
$(ii)\Rightarrow(iii)$ It is obvious.
$(iii)\Rightarrow(i)$ From (iii) it results that

$$\left\|\Phi_m^n(x)\right\|\leq D$$

for all $(m,n,x)\in\Delta\times X$.
Moreover,

$$(m-n+1)\left\|\Phi_m^n(x)v\right\|=\sum_{k=n}^{m}\left\|\Phi_m^n(x)v\right\|\leq$$

$$\leq\sum_{k=n}^{m}\left\|\Phi_m^k(\varphi(k,n,x))\right\|\left\|\Phi_k^n(x)v\right\|\leq$$

$$\leq D\sum_{k=n}^{m}\left\|\Phi_k^n(x)v\right\|\leq D^2\left\|v\right\|$$

for all $(m,n,x,v)\in\Delta\times X\times V$. By Lemma 1 it results that (A,φ) is u.e.s.

Remark 4 *The preceding theorem can be viewed as a Datko-type theorem for the property of uniform exponential stability for discrete evolution semiflows.*

A Barbashin-type theorem for uniform exponential stability of discrete evolution semiflows is given by

Theorem 2 *The following statements are equivalent:*
(i) the system (A, φ) *is uniformly exponentially stable;*
(ii) there are $b > 0$ *and* $B \geq 1$ *such that:*

$$\sum_{k=n}^{m} e^{b(m-k)} \left\| \Phi_m^k(\varphi(k,n,x)) \right\| \leq B$$

for all $(m,n,x) \in \Delta \times X$;
(iii) there exist $b > 0$ *and* $B \geq 1$ *with:*

$$\sum_{k=n}^{m} \left\| \Phi_m^k(\varphi(k,n,x)) \right\| \leq B$$

for all $(m,n,x) \in \Delta \times X$.

Proof. $(i) \Rightarrow (ii)$ If (A, φ) is u.e.s. then there are $N \geq 1$ and $\alpha > 0$ such that for every $b \in (0, \alpha)$ we have

$$\sum_{k=n}^{m} e^{b(m-k)} \left\| \Phi_m^k(\varphi(k,n,x)) \right\| \leq N \sum_{k=n}^{m} e^{(b-\alpha)(m-k)} \leq B$$

for all $(m,n,x) \in \Delta \times X$, where $B = \frac{Ne^{\alpha-b}}{e^{\alpha-b}-1}$.
$(ii) \Rightarrow (iii)$ It is obvious.
$(iii) \Rightarrow (i)$ From (iii) it results

$$\|\Phi_m^n(x)\| \leq B$$

for all $(m,n,x) \in \Delta \times X$. Then

$$(m-n+1) \|\Phi_m^n(x)v\| = \sum_{k=n}^{m} \|\Phi_m^n(x)v\| \leq$$

$$\leq \sum_{k=n}^{m} \left\| \Phi_m^k(\varphi(k,n,x)) \right\| \|\Phi_k^n(x)\| \|v\| \leq B^2 \|v\|$$

for all $(m,n,x,v) \in \Delta \times X \times V$. According to Lemma 1, it results that (A, φ) is u.e.s.

Open problem. If (A, φ) is u.e.s. then there exist $B \geq 1$ such that

$$\sum_{k=n}^{m} \left\| \Phi_m^k(\varphi(k,n,x))v \right\| \leq B \|v\|$$

for all $(m,n,x,v) \in \Delta \times X \times V$. The converse implication is valid?

3 Nonuniform exponential stability

Let (A, φ) be a discrete variational system associated to the discrete evolution semiflow $\varphi : \Delta \times X \to X$ and to the sequence of mappings $A = (A_m)$, where $A_m : X \to \mathcal{B}(V)$, for all $m \in \mathbf{N}$.

Definition 4 *The system (A, φ) is said to be (nonuniformly) exponentially stable (and denote e.s.) if there are three constants $N \geq 1$, $\alpha > 0$ and $\beta \geq 0$ such that:*

$$e^{\alpha(m-n)} \|\Phi_m^n(x)v\| \leq Ne^{\beta n} \|v\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$.

Remark 5 *This concept of nonuniform exponential stability has been introduced in the works of Barreira and Valls (see for example [2]).*

Remark 6 *Using the property (c_2) from Remark 1 it is easy to see that (A, φ) is exponentially stable if and only if there are $N \geq 1$, $\alpha > 0$ and $\beta \geq 0$ with*

$$e^{\alpha(m-n)} \|\Phi_m^p(x)v\| \leq Ne^{\beta n} \|\Phi_n^p(x)v\|$$

for all $(m, n, p, x, v) \in T \times X \times V$.

Remark 7 *It is obvious that*

$$u.e.s. \Rightarrow e.s.$$

The following example shows that the converse implication is not valid.

Example 3 *Let (X, d) be the metric space, V the Banach space and φ the evolution semiflow given as in Example 2.*

We define the sequence of mapings $A_m : X \to \mathcal{B}(V)$ by

$$A_m(x)v = \frac{u(m)}{u(m+1)} e^{-\int_0^1 x(\tau)d\tau} v$$

for all $(m, x, v) \in \boldsymbol{N} \times X \times V$, where the sequence $u : \boldsymbol{N} \to \boldsymbol{R}$ is given by $u(m) = e^{m\pi(1-\cos \frac{m\pi}{2})}$.

We have, according to the definition of discrete evolution cocycle,

$$\Phi_m^n(x)v = \begin{cases} \frac{u(n)}{u(m)} e^{-\int\limits_0^{m-n} x(\tau)d\tau} v, & \textit{if } m > n \\ v, & \textit{if } m = n. \end{cases}$$

We observe that

$$|\Phi_m^n(x)v| = e^{n\pi(1-\cos\frac{n\pi}{2}) - m\pi(1-\cos\frac{m\pi}{2})} e^{-\int\limits_0^{m-n} x(\tau)d\tau} |v| \leq$$

$$\leq e^{2n\pi} e^{-\alpha(m-n)} |v|$$

for all $(m, n, x, v) \in \Delta \times X \times V$*, which prove that* (A, φ) *is e.s.*

Let us suppose now that the system (A, φ) *is u.e.s. Accordind to Remark 3, there exist* $N \geq 1$ *and* $\nu > 0$ *such that*

$$n\pi(1 - \cos\frac{n\pi}{2}) - m\pi(1 - \cos\frac{m\pi}{2}) - \int\limits_0^{m-n} x(\tau)d\tau \leq \ln N - \nu(m - n)$$

for all $(m, n, x) \in \Delta \times X$*. If we consider* $n = 4k + 2$ *and* $m = 4k + 4$*,* $k \in \boldsymbol{N}$ *we have that*

$$8k\pi + 4\pi \leq \ln N + 2x(0) - 2\nu$$

which, for $k \to \infty$*, leads to a contradiction. This proves that* (A, φ) *is not u.e.s.*

A Datko-type theorem for nonuniform exponential stability of variational nonautonomous discrete-time equations is given by

Theorem 3 *The system* (A, φ) *is exponentially stable if and only if there are* $c \geq 0$*,* $d > 0$ *and* $D \geq 1$ *such that:*

$$\sum_{k=n}^{\infty} e^{d(k-n)} \|\Phi_k^n(x)v\| \leq De^{cn} \|v\|$$

for all $(n, x, v) \in \boldsymbol{N} \times X \times V$*.*

Proof. *Necessity.* If (A, φ) is e.s. then there are $N \geq 1$, $\alpha > 0$ and $\beta \geq 0$ such that for $d \in (0, \alpha)$ we have that

$$\sum_{k=n}^{\infty} e^{d(k-n)} \|\Phi_k^n(x)v\| \leq Ne^{\beta n} \sum_{k=n}^{\infty} e^{(d-\alpha)(k-n)} \|v\| = De^{cn} \|v\|$$

for all $(n, x, v) \in \mathbf{N} \times X \times V$, where $c = \beta$ and $D = \frac{N}{1-e^{d-\alpha}}$.
Sufficiency. We observe that from hypothesis it results that

$$e^{d(m-n)} \|\Phi_m^n(x)v\| \leq De^{cn} \|v\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$, which shows that (A, φ) is e.s.

Another characterization of nonuniform exponential stability of variational nonautonomous discrete-time equations is given by

Lemma 2 *The system (A, φ) is exponentially stable if and only if there are $b > c \geq 0$ and $N \geq 1$ such that:*

$$e^{b(m-n)} \|\Phi_m^n(x)v\| \leq Ne^{cm} \|v\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$.

Proof. *Necessity.* If (A, φ) is e.s. then there are $N \geq 1$, $\alpha > 0$ and $\beta \geq 0$ such that:

$$e^{b(m-n)} \|\Phi_m^n(x)v\| = e^{(\alpha+\beta)(m-n)} \|\Phi_m^n(x)v\| \leq$$
$$\leq Ne^{\beta n} e^{\beta(m-n)} \|v\| = Ne^{\beta m} \|v\| = Ne^{cm} \|v\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$, where $b = \alpha + \beta > \beta = c$.
Sufficiency. From hypothesis it results that

$$\|\Phi_m^n(x)v\| \leq Ne^{cm} e^{-b(m-n)} \|v\| =$$
$$= Ne^{cn} e^{-(b-c)(m-n)} \|v\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$.

Finally, we obtain that (A, φ) is e.s.

A Barbashin-type theorem for nonuniform exponential stability of variational nonautonomous discrete-time equations is given by

Theorem 4 *The the system (A, φ) is exponentially stable if and only if there are $b > c \geq 0$ and $B \geq 1$ such that:*

$$\sum_{k=n}^{m} e^{b(m-k)} \left\| \Phi_m^k(\varphi(k,n,x)) \right\| \leq Be^{cm}$$

for all $(m, n, x) \in \Delta \times X$.

Proof. *Necessity.* If (A, φ) is e.s. then by Definition 4 it follows that there are $N \geq 1$, $\alpha > 0$ and $\beta \geq 0$ such that for every $b \in (\beta, \alpha + \beta)$ we have

$$\sum_{k=n}^{m} e^{b(m-k)} \left\| \Phi_m^k(\varphi(k,n,x)) \right\| \leq Ne^{(b-\alpha)m} \sum_{k=n}^{m} e^{(\alpha+\beta-b)k} \leq Be^{cm}$$

for all $(m, n, x) \in \Delta \times X$, where $c = \beta$ and $B = N \frac{e^{\alpha+\beta-b}}{e^{\alpha+\beta-b}-1}$.
Sufficiency. By hypothesis it follows that there exist $B \geq 1$ and $b > c \geq 0$ such that

$$e^{b(m-n)} \left\| \Phi_m^n(x) \right\| \leq Be^{cm}$$

for all $(m, n, x) \in \Delta \times X$. By Lemma 2 it follows that (A, φ) is e.s.

Open problem. If (A, φ) is e.s. then there exist $B \geq 1$ and $b > c \geq 0$ such that

$$\sum_{k=n}^{m} e^{b(m-k)} \left\| \Phi_m^k(\varphi(k,n,x))v \right\| \leq Be^{cm} \|v\|$$

for all $(m, n, x, v) \in \Delta \times X \times V$. The converse implication is true?

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