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In Memoriam Adelina Georgescu

A NEW LOOK AT THE LYAPUNOV INEQUALITY*

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Abstract

Given a Banach space E, it is proved that any function u in $C^2([a,b],E)$ verifies the inequality

$$\max\left\{\|u(a)\|,\|u(b)\|\right\}+\frac{b-a}{4}\int_a^b\|u''(t)\|\,dt\geq\sup_{t\in[a,b]}\|u(t)\|.$$

The constant $(b-a)/4$ is sharp. Several applications are included.

MSC: Primary 26D10, 34B24; Secondary 26A24, 26A45, 46B20.

keywords: Sturm-Liouville problem, function of bounded variation, differentiable function.

1 Introduction

The well-known Lyapunov inequality states that if $q:[a,b]\to\mathbb{R}$ is a continuous function, then a necessary condition for the boundary value problem

$$\begin{cases} u''+qu=0 \\ u(a)=u(b)=0, \end{cases} \tag{1}$$

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