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A NOTE ON METRIC SPACES WITH CONTINUOUS MIDPOINTS*

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Abstract

A metric space (X, d) is a continuous midpoint space if there is a continuous map $\mu : X \times X \to X$ such that, for all $(a, b) \in X \times X$, $d(a, \mu(a, b)) = (1/2)d(a, b) = d(b, \mu(a, b))$. A closed subset C of a complete continuous midpoint space is convex if $\forall (a, b) \in C \times C$, $\mu(a, b) \in C$. Under suitable, but natural, assumptions continuous midpoint spaces are absolute retracts; Browder, Michael or Cellina like continuous selection theorems hold; bounded closed convex sets have the fixed point property for nonexpansive maps. Hyperconvex metric spaces, Cartan-Hadamard manifolds and more generally Hadamard spaces or metric spaces with non positive curvature in the sense of Busemann are continuous midpoint spaces.

MSC: 47H10, 51D99, 53C70, 54C65

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1 Introduction

Given two points a and b of a metric space (X, d) a point m of X is a **midpoint for the pair** $(\mathbf{a}, \mathbf{b})$ if $d(a, m) = (1/2)d(a, b) = d(b, m)$. For all pairs of points of a complete metric space (X, d) to have a midpoint it is

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