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TOEPLITZ OPERATORS WITH BOUNDED HARMONIC SYMBOLS*

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Abstract

In this paper we have shown that if $\phi \in h^{\infty}(\mathbb{D})$ and $T_{\phi}^{(\alpha)}$ is the Toeplitz operator with symbol ϕ defined on the weighted Bergman space $L_a^2(dA_\alpha)$ and if the set $\left\{\left(T_{\phi}^{(\alpha)}\right)^* T_{\phi}^{(\alpha)} f, \left(T_{\phi}^{(\alpha)}\right)^* f, T_{\phi}^{(\alpha)} f, f\right\}$ is linearly dependent for all $f \in L_a^2(dA_\alpha)$ then either ϕ is a constant function or there exists $\lambda_\alpha, \mu_\alpha \in \mathbb{C}$ such that $\frac{\phi-\mu_\alpha}{\lambda_\alpha}$ is a real-valued function in $h^{\infty}(\mathbb{D})$. Here $h^{\infty}(\mathbb{D})$ is the set of all bounded harmonic functions on the open unit disk $\mathbb{D}$.

MSC: 47B38, 47B32

keywords: Weighted Bergman spaces, reproducing kernel, Toeplitz operators, self-adjoint operators, harmonic functions.

1 Introduction

Let $dA(z) = \frac{1}{\pi}dxdy$ be the area measure on the open unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ in the complex plane $\mathbb{C}$. It is normalized so that the area of $\mathbb{D}$ is 1. For $\alpha > -1$, let $L^2(\mathbb{D}, dA_\alpha)$ be the space consisting of all absolutely square-integrable, Lebesgue measurable functions on $\mathbb{D}$ with respect to the measure $dA_\alpha(z) = (\alpha+1)(1-|z|^2)^\alpha dA(z),\ z \in \mathbb{D}$. The measure dA_α is a probability measure on $\mathbb{D}$. Let $L_a^2(dA_\alpha)$ be the subspace of all analytic functions of $L^2(\mathbb{D}, dA_\alpha)$. The space $L_a^2(dA_\alpha)$ is called the weighted

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