CORRELATION OPTICS PARADIGM IN MEASURING COHERENCE AND POLARIZATION OF LIGHT

Oleg V. ANGELSKY[1], Christina V. FELDE[2], Peter V. POLYANSKII[3]

Abstract. *In this survey we represent novel feasibilities provided by correlation optics as one of the versions of "optics of observable quantities" (E. Wolf) in measuring coherence and polarization of optical fields. It is shown by two examples that the introduced approaches are relevant to solving diverse problems connected with the presence of optical singularities (both scalar and vector) in heterogeneous in polarization and incompletely spatially coherent light beams. Namely, we present specific vector singularities arising in partially coherent combined beams and demonstrate interconnections between coherence and polarization in controlling new optical phenomenon referred to as optical currents.*

Keywords: partial coherence, partial polarization, optics of observable quantities, correlation optics.

1. Introduction

In this survey we consider applying the Correlation Optics paradigm for measuring intrinsically interconnected characteristics of light fields, such as intensity, polarization and coherence. Conceptually, all these quantities are derived from the Wolf's coherency matrix [1]. New insight on interconnection of them is accentuated by the novel singular-optical approach [2, 3] predicting existence of important regularities in electromagnetic fields which were early considered as quite random ones. So, phase singularities, viz. 'optical vortices' of scalar (homogeneously polarized), polarization singularities of vector (inhomogeneously polarized) fields, as well as singularities of correlation functions of partially coherent, partially polarized fields constitute specific skeletons, sui generis "bearing structures". Really, knowing the loci and characteristics of singular elements, one can judge on behaviour of a field at its other areas, at least in qualitative manner, but quite reliably [4]. Potentially, this circumstance opens new feasibilities for metrology of optical fields and leads to prospective practical applications of relevant measuring techniques. Developing earlier approaches [5], here we show the framework for generalization

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of polarization metrology for a wide class of combined optical beams assembled from mutually incoherent (or partially mutually coherent) components, which can be even orthogonal in polarization. Such generalization provides taking into account partial polarization and associated specific vector singularities, which can be used in future for non-destructive optical diagnostics as well as in optical telecommunications with polarization data coding. Important part of this survey is devoted to description of the feasibilities for experimental measuring coherence by measuring of spatial polarization distributions of inhomogeneously polarized fields. We represent the newest metrological tool connected with the concept of optical currents (optical flows) [6]. Namely, we show that some intimate characteristics of complex optical fields with arbitrary degree of spatial coherence and arbitrary degree of polarization may be “deciphered” indirectly, by observation of the influence of such fields on embedded micro- and nanoparticles. On the other hand, this metrological approach seems to be prospective for development of so-called optical traps and tweezers for manipulation of isolated particles of micro- and nanoscales that is of vital importance for control of thin films and growing crystals, in pharmacology, precision chemistry, nanophotonics and other applications where one must operate with super-small quantities of matter. Note, the concept, approaches and previous results of Correlation Optics have been popularized in recent issue of Optics and Photonics News [7] in more general framework than it is presented here by consideration in more details of selected instructive examples.

2. Background

The notion of coherence is the most fundamental concept of modern optics. As it has been shown by E. Wolf [1, 8], this notion is intrinsically connected with other characteristics of light, such as intensity and polarization. Really, one can distinguish between these characteristics in didactic purposes, but in every practically important case we meet tight, inseparable interconnection of them. So, one cannot define coherence, in part aspiring to associate it with visibility of interference pattern, ignoring for that the states of polarization of superposed beams. At the same time, the most fundamental definition of polarized light is given just through the measure of mutual coherence of the orthogonally polarized components of a beam [9]. At last, all three mentioned characteristics of a light beam are comprehensively expressed through known combinations of the Wolf’s coherency matrix elements [1].

Incidentally, urge towards to associate coherence just with obvious interference (intensity modulation) effect does not always lead to true understanding the coherence phenomena. It is not enough that interference fringes are absent in superposition of completely mutually coherent but orthogonally polarized beams (it is well-known from the Fresnel-Arago laws and experiments). There are quite new concepts showing the absence of interference effect for superposing two waves of equal frequencies with strictly (*deterministically*) connected complex disturbances even with the same state of polarization. Refined example of this kind was given by L. Mandel in his concept of "anticoherence" [10], see details in [11].

The next, and more closer to our consideration, example – *pseudodepolarization* [12] (in modern terminology, "global" depolarization [13]) resulting from stationary scattering of laser radiation in multiply scattering media, such as turbid media, multi-mode waveguides, the most of natural objects, including biological ones. Here the role of detector (and its spatial resolution) becomes fundamental. Really, the universal approach to determine all polarization characteristics of a field (both the state of polarization and the degree of polarization [14]) consists in Stokes-polarimetry of the analyzed field. For that, Stokes-polarimetric analysis gives quite different results for local and "global" (space-averaged) measurements. So, the point-wise measuring Stokes parameters shows complete (unity) degree of polarization, but the state of polarization changes from point to point. Space averaging over ten and more speckles shows seeming depolarization. This case is the central subject of interest of vector singular optics [15].

One more example concerns optical currents (flows) [6]. Though it is prematurely now to solve comprehensively this problem, especially in experimental aspect, it is clear that micro- or nanoparticles serving for diagnostics of inhomogeneously polarized and partially coherent optical field [16-18] affect this field as absorbing and retransmitting particles with their own characteristics, so that the state of a field, in general, changes under influence of such secondary radiators.

Pronouncing call of the times in the topic under consideration consists in involving the ideas, approaches and techniques of Singular Optics [2]. It is seen, in part, from recent important review [3] devoted to the structure of partially coherent optical fields. As it has been argued in papers [18, 19], "*Usual beam parameters either characterize a beam 'in a whole' (power, momentum, beam size and divergence angle) or describe its 'shape' via certain spatial distributions (amplitude, phase, polarization state, etc.)… Usual beam parameters provide only rough and, sometimes, distorted picture of internal processes that constitute a real 'inner life' of a light beam. These processes are related to the fundamental dynamical and geometrical aspects of light fields, and are associated with the permanent energy redistribution inside the beam 'body', which underlies the beam evolution and transformations. The internal energy flows provide a natural and efficient way for 'peering' into the light fields and studying their most intimate and deep features*." It is of interest to correlate this statement with the Wolf's methodology of *observable quantities* that is the most influential concept of physical optics since 1954: "optics of observable quantities, such as correlation functions and averaged in time intensities" [14] that has found many henchmen [20-25]. Paradoxical contradiction between two undoubtedly true statements is apparent. Really, this contradiction is just eliminated as one takes into account that internal energy flows may be revealed only by carrying out the experiments with observable quantities.

3. Polarization singularities in partially coherent light beams

In this section we describe specific polarization singularities arising in incoherent superposition of coaxial orthogonally polarized laser beams. It is shown that in transversal cross-section of paraxial combined optical beams of this class, instead of common singularities, such as amplitude zeroes (optical vortices) inherent in scalar fields [2], and polarization singularities such as C points and L lines inherent in completely coherent vector fields [15], *phase singularities of the complex degree of polarization* (CDP) arise, whose description and investigation have been initiated by papers [26-29] basing on earlier studies [30-32] concerned to the Young's concept of the edge diffraction wave in connection with diagnostics of phase singularities of spatial correlation functions of optical fields. There are U contours along which the degree of polarization equals zero and the state of polarization is undetermined (singular), and isolated P points where the degree of polarization equals unity and the state of polarization is determined by the non-vanishing component of the combined beam.

Let us briefly argue the relevance of the introduced approach. In scalar fields, when polarization can be neglected, so-called screw wave front dislocations (also referred to as amplitude zeroes or optical vortices) take place. Phase of the complex amplitude is undetermined at such elements and is step-like changed at crossing of them. In vector fields optical vortices are absent, though they remain in any polarization ("scalar") component. Instead of vortices, polarization singularities arise at cross-section of a field, *viz.* field elements where azimuth of polarization (C points) or handedness (L lines) is undetermined [15]. Vector skeletons of coherent inhomogeneously polarized fields were elaborated in details in papers [33-35]. By crossing L lines, handedness is step-like changed into opposite one; by crossing C point, azimuth of polarization is changed into orthogonal one. Mentioned singularities disappear in the case of partially coherent wave fields (though they remain in each completely coherent component, mode in a set of which partially coherent radiation is decomposed. Instead of them, new singularities appear inherent just in partially coherent fields. Singularities of partially coherent fields have formed the novel topic in the field of singular optics just at the beginning of the Third Millenium [3].

For that, two situations arise again: (i) scalar case when polarization can be ignored while the state of polarization is the same at all point of a field, and (ii) vector case when the state of polarization of *partially coherent* field changes from point to point that requires explicit taking into account of vector nature of light. The first (scalar) case became the subject of intense investigations in last years [31, 36, 37]. As a result of these investigations, new phase singularities of spatial and temporal correlation functions of quasi-monochromatic light fields have been revealed, as well as singularities of spectral components of polychromatic ("white-light") radiation [32, 38-41].

3.1. U and P singularities in partially spatially coherent combined beams

Let us consider vector singularities in partially coherent optical beams by giving the following simple instructive example. Mutually incoherent and orthogonally polarized Laguerre-Gaussian mode LG01 and a plane wave are coaxially mixed. Such components can be obtained from one laser (using a computer-generated hologram for forming doughnut LG01 mode) in interferometric arrangement with optical delay, Δl, considerably exceeding a coherence length of the used laser, l, or using two different lasers. Intensity of a plane wave is set deliberately to be less than the peak intensity of the mode, see Fig. 1.

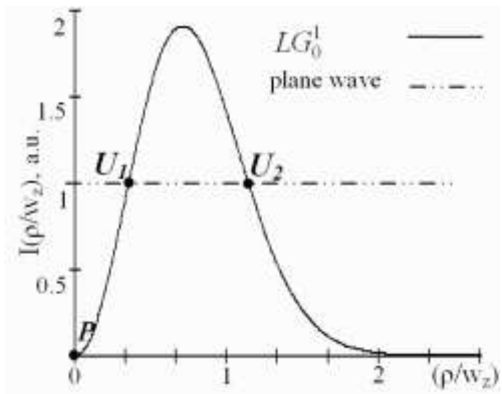


Figure 1. Radial intensity distribution of mixing vortex-supporting LG01 mode and plane wave with intensity less than peak intensity of a mode in function of dimensionless radial variable ρ/w_z .

Thus, we consider two-component mixture of co-directional orthogonally polarized beams, one of which contains a 'scalar' phase singularity, *viz.* optical vortex. Interference between such beams with forming common interference fringes is excluded by two reasons: (i) specified mutual incoherence of the components: (ii) polarization orthogonality of them. Note, even only the second condition *per si* determines that, independently on the degree of mutual coherence of two beams over whole interval from zero (for optical path difference exceeding the coherence length) to unity (for zero optical path difference) *visually* observed and *photometrically* measured pattern remains unchangeable. However, more delicate polarization analysis of the combined beam enables to differentiate two limiting cases, *viz.* completely coherent and completely incoherent mixing of orthogonally polarized components.

Let us firstly consider the limiting case when two components are completely mutually coherent. For the sake of distinctness (and for substantiveness of further consideration), we consider coherent mixing of orthogonally *circularly* polarized LG01 mode and a plane wave. Beside of all, choice of circular polarization basis possesses the advantage of invariance in respect to rotation of the coordinates, in contrast to linear or elliptical bases [42].

In general, combined beam, everywhere with the unit polarization degree ($\sqrt{s_1^2 + s_2^2 + s_3^2} = 1$, where s_1, s_2, s_3 are the normalized second, third and fourth Stokes parameters, respectively [9, 42], is elliptically polarized. But at the center of vortex of LG01 mode the field is circularly polarized with the state of polarization of a plane wave. A common phase singularity (vortex) of orthogonally polarized component of the combined beam lies at the bottom of this circular polarization. At the same time, the resulting field is polarized linearly at two contours where amplitudes of two components become equal to each other, see Fig. 2 a.

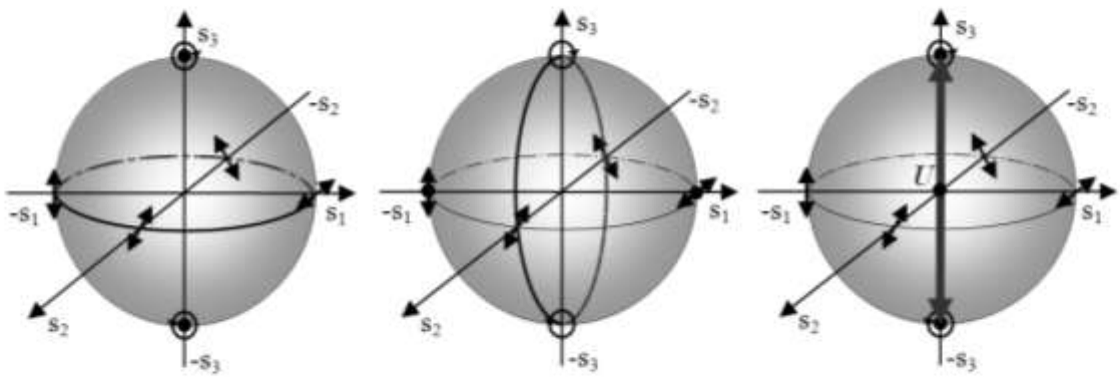


Figure 2. The lines of equal intensities of orthogonally polarized beams at Stokes space:
a. Equator of the Poincare sphere for circular polarization basis, coherent mixing.
b. 45°-meridian including the poles for linear polarization basis coherent mixing.
c. Diameter of the Poincare sphere connecting the poles for circular polarization basis, incoherent mixing

For that, owing to helicoidal structure of a wave front of LG01 mode, azimuth of linear polarization changes with changing phase difference of a mode and a plane wave. Such topological structure can be considered as elementary experimental model of the assemblage of C point and surrounding it L contour of conventional singular optics of vector fields. Really, crossing L line where handedness is undetermined is accompanied by step-like changing handedness into opposite one, corresponding to predominant in intensity component with unchangeable azimuth of polarization. For comparison, Fig. 2 b illustrates the line of equal intensities of coherently mixed components in linear polarization basis. It is of interest that the elementary structure shown in Fig. 2 a is directly related with description of *completely polarized* light at the circular complex polarization plane that is a stereographic projection of the Poincare sphere [43]. So, C point and L contours correspond to the pole of the Poincare sphere and its equator, see Fig. 3 a.

Let us support this intuitive consideration by formal description. Let us proceed from Jones vectors of two components, right-circularly polarized LG01 mode and left-circularly polarized plane wave,

$$\mathbf{E}_{LG} = c(w/\rho)\exp(i\Delta)\begin{bmatrix}\exp(i\varphi)\\ \exp[i(\varphi+\pi/2)]\end{bmatrix}, \qquad \mathbf{E}_P = \begin{bmatrix}\exp(i\varphi)\\ \exp[i(\varphi-\pi/2)]\end{bmatrix}, \tag{1}$$

where c is the amplitude factor corresponding to inhomogeneous amplitude distribution of a mode as a function of dimensionless radial coordinate, and $\exp(i\Delta)$ is associated with helicoidal change of a phase of a mode under circumference of the central vortex (its explicit form for Laguerre-Gaussian mode is well known but is not relevant here). There is Jones vector of the combined beam:

$$\mathbf{E}_{Total} = \mathbf{E}_{LG} + \mathbf{E}_P = \begin{bmatrix}E_x\\ E_y\end{bmatrix} = \begin{bmatrix}c\exp(i\Delta)+1\\ c\exp[i(\Delta+\pi/2)]+\exp(-i\pi/2)\end{bmatrix}\exp(i\varphi) \tag{2}$$

General coherency matrix of the beam is found as

$$\{\mathbf{J}\} = \mathbf{E}_{Total}\cdot\mathbf{E}^*_{Total} = \begin{bmatrix}E_x\\ E_y\end{bmatrix}\begin{bmatrix}E^*_x & E^*_y\end{bmatrix} = \begin{Vmatrix}J_{xx} & J_{xy}\\ J_{yx} & J_{yy}\end{Vmatrix}, \tag{3}$$

or in explicit form:

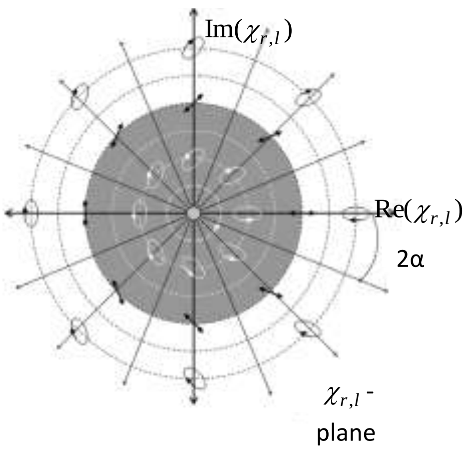
$$\{\mathbf{J}\} = \begin{Vmatrix}c^2+2c\cos\Delta+1 & c^2\exp(-i\pi/2)+2c\sin\Delta+\exp(i\pi/2)\\ c^2\exp(i\pi/2)+2\sin\Delta+\exp(-i\pi/2) & c^2-2c\cos\Delta+1\end{Vmatrix} \tag{4}$$

Combining the elements of coherency matrix, one can find *full* Stokes parameters:

$$S_0 = J_{xx} + J_{yy} = 2(c^2 + 1); \quad S_1 = J_{xx} - J_{yy} = 4c\cos\Delta;$$
$$S_2 = J_{xy} + J_{yx} = 4c\sin\Delta; \quad S_3 = i\left(J_{xy} - J_{yx}\right) = 2(c^2 - 1). \tag{5}$$

Here we are especially interested in the case when $c = 1$. One just obtains for this case the *normalized* Stokes parameters:

$$s_0 = 1; \quad s_1 = \cos\Delta; \quad s_2 = \sin\Delta; \quad s_3 = 0. \tag{6}$$



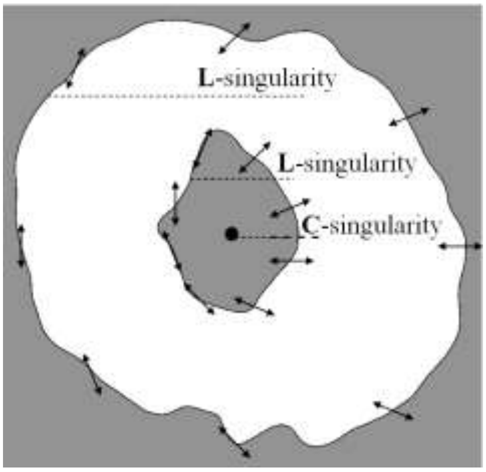


Figure 3.

a. Poincare representation:
The complex circular polarization plane. The center of coordinates corresponds to left-circular polarization (C point); the circle of unite radius separating grey and whit areas corresponds to linear polarizations with changeable azimuth of polarization (L contours), this contour separates the area of the beam with left handedness and right handedness; right-circular polarization point lies at infinity.

b. Experimental:
C and L singularities in combined beam assembled from completely mutual coherent orthogonally (circularly)polarized LG01 mode and plane wave. At L lines, where intensities of two mixed component are equal, the azimuth of polarization changes in agreement of prediction illustrated in Fig. 3 a. Areas of different colors correspond to opposite handedness.

Vanishing of the fourth Stokes parameter means that polarization at all points of the contour where intensities of the mixed components are equal to each other are equally distanced from the states of polarization of the components, i.e. neither right-circular nor left-circular predominate in intensity. It is in direct correspondence with Fig. 2 a. At all points of such L contour polarization is linear with the polarization azimuth $\alpha = 0.5\tan^{-1}(s_2/s_1) = \Delta/2$, while the angle of ellipticity $\beta = 0.5\arcsin s_3 = 0$. For that, the degree of polarization $P = \sqrt{s_1^2 + s_2^2} \equiv 1$. In correspondence with helicoidal structure of a wave front of LG01 mode, a phase difference of the components changes along the contour of equal intensities that results in changing azimuth of polarization. Thus, we obtain direct analog of L contour. Further, at the center of vortex of LG01 mode we have $c = 0$. Again, proceeding from Eq. (5) we find the normalized Stokes parameters $1, 0, 0, -1$, i.e. left-circular polarization of a plane wave. In the vicinity of such C point polarization is elliptical, with the azimuth of polarization changing with azimuthal coordinate and ellipticity decreasing from the vortex to L contour, Eq. (6), where handedness is undetermined an step-like changing by crossing this contour. It is all in quite correspondence with Fig. 3 a.

Thus, for circular polarization basis, walking along contour of the combined beam “LG01 mode + plane wave” where intensities of the components become equal corresponds to moving along equator of the Poincare sphere that is determined only by the ratio of the second and third Stokes parameters. (For comparison, using linear polarization basis, to say 0° and 90°, one obtains by the same way the normalized Stokes parameters for the combined beam $1, 0, \cos[\Delta + (\varphi_0 - \varphi_{90})]$, $\sin[\Delta + (\varphi_0 - \varphi_{90})]$ that corresponds to points of 45°-merdian of the Poincare sphere, see Fig. 2 b.)

Before consideration of the most general case of partial mutual coherence of the mixed orthogonally polarized components in the following section, let us consider other limiting case, *viz.* completely incoherent mixing of such components. There is no necessity to proceed now from Jones vectors and to form a coherency matrix of the combined beam. One can at once determine the Stokes parameters of mutually incoherent components and sum them directly, without accounting phase relations that are irrelevant for incoherent summation. The normalized Stokes parameters of orthogonally polarized beams differ only in sign of the second, third and fourth parameters: $\{1, s_1, s_2, s_3\}$ and $\{1, -s_1, -s_2, -s_3\}$. It is clear that when two components become equal in intensities, the normalized Stokes parameters of the combined beam becomes $\{1, 0, 0, 0\}$. The field at such elements of a field is completely unpolarized. There are just U singularities [26-28]. This case is shown in Fig. 2 c for the case of incoherent mixing of orthogonally circularly polarizer components. (Note, this case *can not be* directly reflected at the complex polarization plane, Fig. 3 a, but *can be* imaged in a whole Stokes space bounded by the Poincare sphere!) Trajectory of the imaging point for the combined beam in this case is the diameter of the Poincare sphere connecting two poles. U singularity is imaged by the center of this sphere, and all other points of this diameter (beside the center and poles) image partially circularly polarized states. For that, the length of a vector drawn from the center of the Poincare sphere to the imaging point inside it equals the degree of polarization. The point where the degree of polarization equals unity is referred to as P (completely polarized) point [26, 27]. Its location is determined by the vortex of orthogonally polarized (scalar singular) component. The set of P points and U contours corresponding to extrema of the degree of polarization of a field are *the singularities of the degree of polarization* forming the vector skeleton of two-component mixture of orthogonally polarized beams. Note, in papers [26-28] consideration is carried out using the notion of the complex degree of polarization – CDP, associated with orientation of the vector of polarization in the Stokes space and undergoing the phase singularity at the center of this space. So, U singularities can be considered just as vector singularities, *viz.* singularities of the vector of polarization, when its magnitude equals zero and a phase (orientation of the vector) is undetermined.

Let us emphasize that the condition of occurring U singularity (equalizing intensities of orthogonal circular components) is equal to the condition of occurring L contour in completely coherent limit. It means that loci of C and L singularities in completely coherent fields and P and U singularities in partially coherent fields arising from completely incoherent orthogonally polarized components, correspondingly, coincide.

Moving from U singularity results in predomination of one of two orthogonal components in intensity. The state of (partial) polarization is just determined by the predominant component. That is why, the degree of polarization can be determined in similar form as visibility:

$$P = \left| \frac{I_1 - I_2}{I_1 + I_2} \right| . \qquad (7)$$

In other words, at each point of the combined beam equal in intensities parts of orthogonal components form unpolarized background, at which completely polarized part corresponding to predominant in intensity component manifests itself. This is in a complete agreement with classical decomposition of partially polarized beam into completely coherent and completely incoherent parts, which are added on intensities, without accounting phase relations [9, 14]. Note, there are no any device providing such decomposition in practice. However, share of completely polarized part can be determined experimentally through the Stokes polarimetric experiment, $P = \sqrt{s_1^2 + s_2^2 + s_3^2}$ or, equivalently, following Eq. (7). Thus, only two orthogonal states of polarization take place in combined beams of considered kind, which are separated by U singularities where the state of polarization is undetermined.

So, the considered limiting cases show the same location of C and P singularities and L and U singularities for the same set of components. However, vicinities of such singularities are essentially different. Only two orthogonal states of polarization are present in spatially partially coherent combined beams, and only the degree of polarization changes from point to point within the areas separated by U singularities.

3.2.Vector singularities for partially mutually coherent mixed components

Let us consider now the most general case, when two mixed components shown in Fig. 1 are orthogonally (circularly) polarized and are partially mutually coherent, so that the degree of mutual coherence of the components can be gradually changed from unity to zero. It can be implemented in the arrangement of the Mach-Zehnder interferometer with controllable optical pass difference between the legs of an interferometer, see detail description of practical arrangement in [9, 28]. Namely, one controls path delay Δl from zero to magnitude exceeding a coherence length (length of wave train) l of the used laser. Change of the ratio $\Delta l/l$ corresponds to change degree of mutual coherence of orthogonally polarized components. Thus, for $0<\Delta l/l<1$ the combined beam is *simultaneously* partially spatially coherent (due to changing intensity ratio at cross-section of the resulting field) and partially temporally coherent (due to non-zero optical path difference between the components), one expects for increasing optical path difference the following.

As it has been mentioned above, the condition of arising of L contours and U contours in the limiting cases of mixing of orthogonally circularly polarized beams is the same: intensities of the components must be equal to each other. If the optical path difference increases from zero, field at the L contour remains linearly polarized, but the degree of polarization decreases. It follows from that the degree of polarization of a beam is determined by the degree of mutual coherence of its arbitrary orthogonal components, here right-hand and left-hand circular components. It means that *U contour nucleates just at the bottom of L contour.*

The degree of polarization can be represented equivalently in terms of *measured* Stokes parameters (that will be used in the next section) or *theoretically*, *viz.* through the invariants of the coherency matrix, which at the same time determine coherence properties of a field [1]:

$$P=\sqrt{1-\frac{4\det\{\mathbf{J}\}}{\mathrm{Sp}^2\{\mathbf{J}\}}}\,. \tag{8}$$

For that, in general, the degree of polarization is always non less than modulo of the degree of mutual coherence of the components, for circularly polarized components

$$|\mu_{rl}| = \left| \frac{J_{rl}}{\sqrt{J_{rr} J_{ll}}} \right| \tag{9}$$

In general case $P \neq |\mu_{rl}|$, as the degree of coherence depends on the decomposition basis while the degree of polarization is invariant [1]. However, it has been shown [1] that the degree of polarization is equal to the *maximal* degree of coherence, $P \equiv |\mu_{rl}|_{\max}$, in the case when the components are of equal intensities. This is just the case of L singularities and U singularities. It is of the most importance, that change of the optical path difference changes weights $|\mu_{rl}|$ of completely coherent (and completely polarized) part of the combined beam and $1-|\mu_{rl}|$ of its completely incoherent part. Increasing $\Delta l/l$ difference corresponds to increasing weight of U singularity against L singularity, so that one can follow gradual transformation of L contour into U contour.

3.3. Experimental reconstruction of "pure" and "mixed" polarization singularities

Mixing of orthogonally circularly polarized LG01 mode and plane wave was performed [26, 27] for intensity of a plane wave less than the peak intensity of a mode approximately by the order of magnitude. The following results have been obtained under such conditions.

Fig. 4 a shows the combined beam whose view, as was mentioned above, within experimental accuracy remains the same at arbitrary optical path delay set in the interferometer.

Figure 4.

a. The partially coherent combined beam.

b. Vector skeleton of the partially coherent combined beam formed by P and U singularities for completely incoherent mixing of circularly polarized components. Compare with Fig. 3 b.

This photo has been obtained for incoherent mixing of two components for $\Delta l/l \approx 3$ (under condition realized in paper [31]). We measured spatial distribution of the Stokes parameters and looked for the elements where $s_1 = s_2 = s_3 = 0$ ($P = 0$, U contours), and $s_3 = 1$ (P point), see discussion following Eq. (6). In such a manner, we were in a position to reconstruct a vector skeleton of partially spatially coherent combined beam formed by completely mutually incoherent components. Experimental error in determining the normalized Stokes parameters was at the level 7%; this determines reliability with which we reconstructed P point and U contours. P and U singularities for this case are shown in fragment of Fig. 4 b. Two U contours separate the areas with right-circular and left-circular polarization shown by different levels of grey. Within these areas $\sqrt{s_1^2 + s_2^2} = 0$, while $s_3 < 1$.

Separate maps of the Stokes parameters are less representative being only row material for finding out the degree of polarization, ellipsometric parameters of a field, and vector singularities. That is why, we demonstrate separately from 2D pattern shown in Fig. 5. 1D cross-section of the degree of polarization of this combined beam, see Fig. 1 b. Dashed curve shows two-lateral radial dependence of P computed following Eq. (7). Solid curve shows experimentally obtained distribution found as the combination of measured Stokes parameters, here $P = |s_3|$. Quantitative discrepancy of two curves (both in positions of zeroes and in heights of side-lobes) is explained by anisotropy of the vortex. Nevertheless, the experimental dependence is in satisfactory qualitative agreement with the simulation results. Namely, one observes two zeroes of the CPD at the each side of the central optical vortex that are the 'fingerprints' of two U contours. Moreover, experiment has proved typical conical vicinity of U contours [26], which are reliable sign of true singularity of any kind, in contrast to local minimum.

Another limiting case (completely mutually coherent components) for $\Delta l/l << 1$ (approximately 0.05) is illustrated in Fig. 3 b. Again, spatial maps of the Stokes parameters were obtained and the elements $s_3 = 0$ and $\sqrt{s_1^2 + s_2^2} = 1$ where selected. There are the lines of linear polarization. Than, in several selected points of such L lines we determined the azimuth of polarization, again, by two ways: firstly as $\tan^{-1}(s_2/s_1)$ and, secondary, as direct measurement of the azimuth of polarization by rotating a linear analyzed up to complete extinction of a field at the specified point that corresponds to crossed azimuth of polarization of the combined beam and the axis of maximal transmittance of analyzer. Description between two results for determining the azimuth of polarization do not exceeded 0.1 rad.

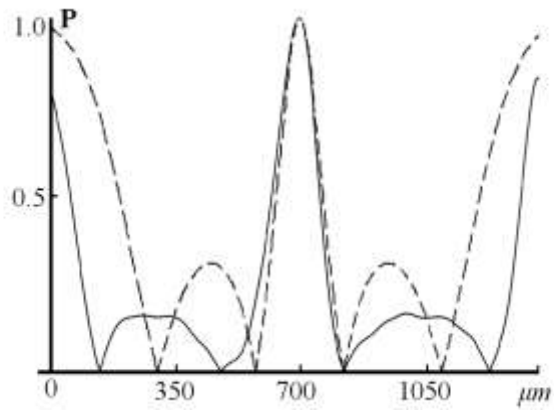


Figure 5. 1D distribution of the degree of polarization of the combined beam formed by two mutually incoherent orthogonally polarized components defined in Fig.1 and shown in Fig. 4.

Perfect extinction of a beam at the specified points just shows that the degree of polarization $P=1$ (in contrast to the case of completely mutually incoherent components, where intensity at the analyzer output is independent on its orientation). Also, for certain orientations of a quarter-wave plate and analyzer, the field at each other point can be extinguished that shows that everywhere the degree of (elliptical) polarization equals unity. It is worth to compare Fig. 3 b with a view of the circular complex polarization plane (Fig. 3 a) to see that, really, such polarization distribution over of a combined beam is close experimental analogue of the circular polarization plane.

At last, we have elaborated experimentally intermediate case, when $0<\Delta l/l<1$, lying between ones considered above. For step-by-step increasing optical path difference between the same orthogonally (circularly) polarized components, we obtained spatial distributions for the Stokes components $I_0, I_{90}, I_{+45}, I_{-45}, I_r, I_l$ and found from them the Stokes parameters. Further, the degree of polarization and ellipsometric parameters of the combined beam were determined as the combinations of these parameters.

Before formulating the conclusions from our observations, let us represent one of row (intermediate) results undergoing following processing. Fig. 6 illustrates combined beams “LG01 mode + plane wave” (with large intensity ratio, so that one does not visualizes a plane wave) for relative optical path differences close to unity (coherent limit) and slightly exceeding a half of the coherence length of used laser, left fragments of Fig. 6. Other fragments of this figure are the intensity distributions I_{+45} (central column) and I_{-45} (right column) used for forming the third Stokes parameters.

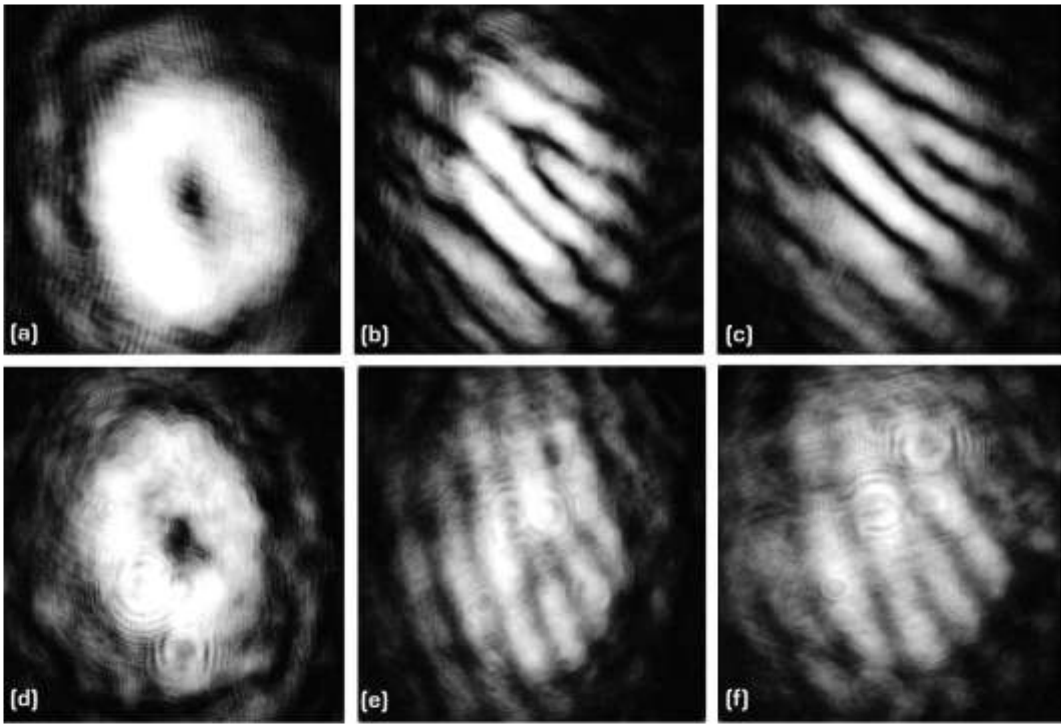


Figure 6.

a, d. The combined beams "LG01 mode + plane wave" with relative optical path differences $\Delta l/l \approx 0.05$ and $\Delta l/l \approx 0.56$, respectively

b, e. The corresponding intensity distributions behind a linear analyzer for determining the third Stokes parameters: +45°

c, f. The corresponding intensity distributions behind a linear analyzer for determining the third Stokes parameters: –45°

Decreasing visibility of interference fringes in fragments e and f corresponds to decreasing in parallel the degree of mutual coherence of the mixed components and the degree of polarization of the combined beam.

Though two orthogonally polarized components do not interfere, their equal polarization projections selected by properly oriented polarizer *can* interfere depending on their mutual coherence. If the degree of mutual coherence of the components is not zero, their equally polarized projections interfere with forming typical patterns indicating phase singularity. Comparison of the central and left columns of Fig. 6 shows that spatial intensity distributions for orthogonal polarization projection of the combined beam are complementary in a sense that dark forklet is replaced by bright one.

The main conclusion follows from comparison of fragments b and e (c and f). Decreasing the mutual coherence of the mixed components and decreasing the degree of polarization of the combined beam are accompanied by decreasing ability of equal polarization projections of the mixed components to interfere that manifests itself in decreasing visibility of interference pattern. So, in fragments b and c of Fig. 6 ($\Delta l/l \approx 0.05$) the measured visibility is 0.97, while in the fragments Fig. 6 e and f ($\Delta l/l \approx 0.56$) visibility is 0.24 (with experimental error non exceeding 5%). It shows the feasibility allows determine *the degree of mutual coherence* of two orthogonally polarized beams by measuring *the degree of polarization* of the combined beam formed by such components found from Stokes parameters. Namely, in our experiment $|\mu_{rl}|$ for $\Delta l/l \approx 0.56$ also equals 0.24. Such measurements are preferably be performed at the elements of the combined beam where intensities of two beams are equal to each other (where L and U singularities co-exist in case of partial mutual coherence of the components), while at such singular elements of the combined beam $P \equiv |\mu_{rl}|$.

Thus, vector singularities occurring in light fields, which are simultaneously partially spatially and partially temporally coherent have been considered in this section. It has been shown that in the case of partially coherent mixing of two orthogonally *circularly* polarized components conventional vector singularities, *viz.* C points and L lines submerged in a field of elliptical polarizations coexist with singularities arising just in partially coherent fields, such as U and P singularities as the extrema of the degree of polarization. Gradual transformation of C and L singularities into P and U singularities, respectively, accompanying decreasing degree of mutual coherence of the components has been experimentally shown. So, conventional polarization singularities of completely coherent fields (C points and L lines) are vanish in incoherent part of the combined beam, so that the only polarization of the component predominant in intensity remains in the vicinities of P points and U lines.

4. Optical currents in completely coherent and partially coherent vector fields

In this section we present the results on the spatial distribution of the Poynting vector governing motion of nanoparticles in spatially inhomogeneously polarized fields.

The Poynting vector $\mathbf{S}$, is defined, in its simplest (Abraham's) form, as the vector product of the vectors of electric and magnetic fields, *viz.* $\mathbf{S} = \mathbf{E} \times \mathbf{H}$. By definition, the Pointing vector *of a plane wave* is perpendicular to vectors $\mathbf{E}$ and $\mathbf{H}$, being representing the energy flux (in W/m^2) of an electromagnetic field. As it will be seen from the following consideration, in light fields with complex wave fronts and with variable in space state of polarization the Poynting vector can exhibit much more sophisticated behavior, being changing from point to point at the beam cross-section and leading to new applications of light.

The use of small particles for diagnostics of microstructure of light is widely used approach [16-18], but mainly in approximation of complete coherence of an optical field. Here we analyze the influence of phase relations and the degree of mutual coherence of superposing waves in two-wave and four-wave configurations on the characteristics of the nanoparticle's motion. The possibility of diagnostics of optical currents in liquids caused by polarization characteristics of an optical field alone is demonstrated using nanoscale metallic particles. We also discuss the prospects of studying temporal coherence using the proposed approach.

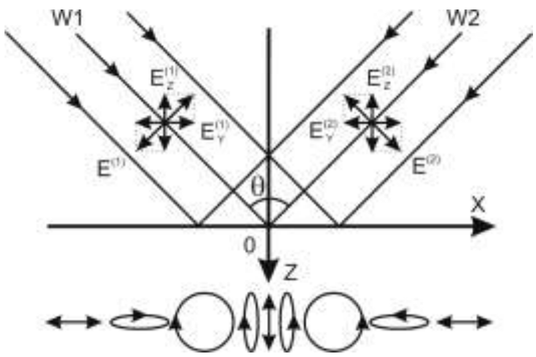
Experimental investigation and computer simulation of the behavior of small spherical particles embedded in optical fields provide a deeper understanding of the role of the Poynting vector for description of optical currents in various media [6, 19]. So, interference between waves polarized in the plane of incidence has been shown to be effective in creation of polarization micro-manipulators; on the other hand, this is a vital step in optimal metrological investigation of optical currents in vector fields [43-46]. Besides, the study of spatial and temporal peculiarities of the motion of particles embedded in optical fields with various spatial configurations and with various scale distributions of the Poynting vector leads to new techniques for estimating the temporal coherence of optical fields.

Computation of the spatial distribution of the time-averaged Poynting vector determining the forces affecting nanoparticles and their movement is performed following the algorithm proposed by M. Berry [6] who has shown that the vector force affecting a small particle in an optical field is proportional to the time-averaged Poynting vector. We will show here that the study of the motion of nanoparticles in inhomogeneously polarized fields provides reconstruction of the spatial distribution of the time-averaged Poynting vectors, *viz.* the optical currents.

4.1. Two-wave superposition for changeable degree of mutual coherence of the components

Superposition of two plane waves of equal amplitudes polarized in the plane of incidence (Fig. 7a) results in the distribution of the Poynting vector shown in Fig. 7b. Such distribution arises when the interference angle is equal to 90°, and the only periodical polarization modulation of the field (in the absence of intensity modulation) takes place in the plane of observation [45].

Analysis of the spatial distribution of the time-averaged Poynting vectors shown in Fig. 7b reveals the periodicity of this distribution, where the lengths of lines shown in the figure are proportional to the absolute magnitudes of the vectors. The lines corresponding to the singularities of the Poynting vector are shown by the indicated set of points. Spatial distribution of the time-averaged Poynting vectors shows the trajectories of energy transfer. The points at the map of the time-averaged Poynting vectors correspond to the areas through which energy transfer is absent, showing: (i) the loci of singularities of the Poynting vector; (ii) the directions along which light energy is non-vanishing (but is conserved); (iii) the points where the vector **H** vanishes due to interference, while in 90°-arrangement vectors **H** of equal amplitudes associated with two plane waves are parallel.



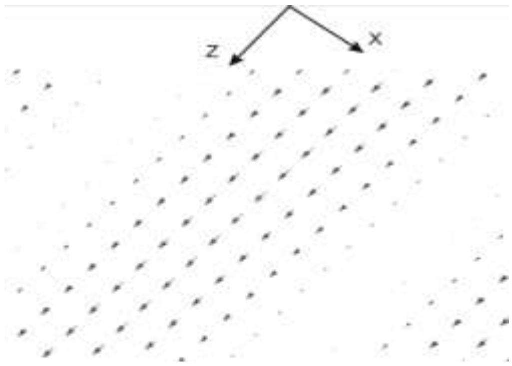


Figure 7.

a. Superposition of plane waves of equal amplitudes linearly polarized in the plane of incidence having an interference angle of 90°. Periodical spatial polarization modulation takes place in the plane of incidence.

b. Spatial distribution of the time-averaged Poynting vectors resulting from superposition of two orthogonally linearly polarized waves with an interference angle of 90°.

The instantaneous magnitude of the electric (magnetic) field strength's vector of the resulting distribution formed in the plane of observation is written as $\mathbf{E} = \left|\mathbf{E}^{(1)} + \mathbf{E}^{(2)}\right| \cos(\omega t + \delta_e)\mathbf{a}_e$ (or $\mathbf{H} = \left|\mathbf{H}^{(1)} + \mathbf{H}^{(2)}\right| \cos(\omega t + \delta_h)\mathbf{a}_h$), where, $\mathbf{a}_e$, $\mathbf{a}_h$ are the unit vectors in the direction of propagation of the electric (magnetic) components for the resulting field in the plane of observation; δ_e (δ_h) is the phase difference of the electric (magnetic) field components of superposed waves. Thus, the instantaneous magnitude of the Poynting vector is

$$\mathbf{S}_{inst} = \mathbf{E} \times \mathbf{H} = |\mathbf{E}| \cdot |\mathbf{H}| \cos(\omega t + \delta_e) \cos(\omega t + \delta_h)(\mathbf{a}_e \times \mathbf{a}_h),$$

and the time-averaged Poynting vector magnitude is

$$\mathbf{S}_{ave} = \frac{|\mathbf{E}| \cdot |\mathbf{H}|}{2} (\mathbf{a}_e \times \mathbf{a}_h) \cos(\delta_e - \delta_h) = \frac{1}{2} (\mathbf{E} \times \mathbf{H}) \cdot \cos(\delta_e - \delta_h). \tag{10}$$

Because the phase difference of the electric field changes from point to point (polarization modulation), the time-averaged magnitude of the Poynting vector is modulated in space taking the maximum (minimum) at different points of the plane of observation, as it is seen from Eq. (10).

Periodical spatial modulation of the Poynting vector in the observation region have previously been discussed [47, 48]. Spatial polarization modulation at the plane of observation is caused by superposition of the E_x and E_z field components with changing the phase difference from point to point, cf. Fig. 7a. A photodetector registers only intensity, $I = E_x^2 + E_z^2$. The sum of the squared amplitudes of the electrical field components is constant at the plane of observation, but the state of polarization changes.

One observes the dependence of the result on the phase relation between vectors $\mathbf{E}$ and $\mathbf{H}$ through the vector magnitude and its direction. This relation changes from point to point in the plane of observation that manifests itself in polarization modulation. Both the magnitudes of projections E_x and E_z and their phases change from point to point. As a consequence, the Poynting vector also changes, see Fig. 8.

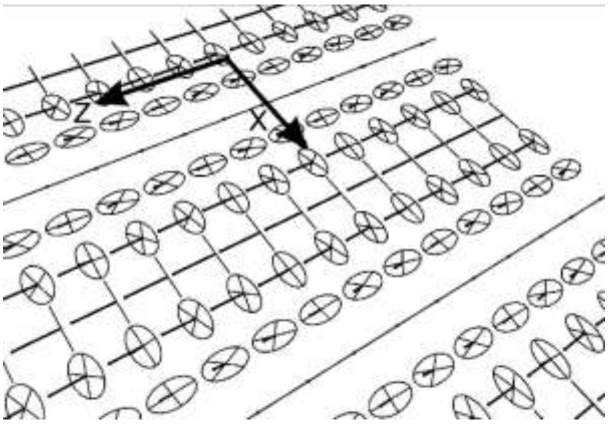


Figure 8. The polarization distribution in the registration plane is marked by thin lines. The direction and magnitude of the Poynting vector are marked by bold lines. The point at the end of the vector determines the energy transfer direction. The modulation of the Poynting vector takes place according to the polarization modulation at the plane of observation.

The results of simulating the motion of particles embedded in the field of the considered distribution of the Poynting vector are shown in Fig. 9. It is tacitly assumed the particles to be absorbing and of size 0.1 μm. One observes that in the case of the distribution resulting from superposition of completely mutually coherent waves, the velocities of particle motion along the lines of maxima and zeroes of the Poynting vector are considerably different from one another.

The particle size is here comparable with a half-period of the corresponding distribution; however, the resultant force giving rise to the particle motion along the lines close to the Poynting vector maxima exceeds the resultant force for lines close to the zeroes of the Poynting vector. The results of modulation of particle movement velocity along the peaks and zeroes of the field of the averaged Poynting vector are shown in Fig. 9a and Fig. 9b, respectively.

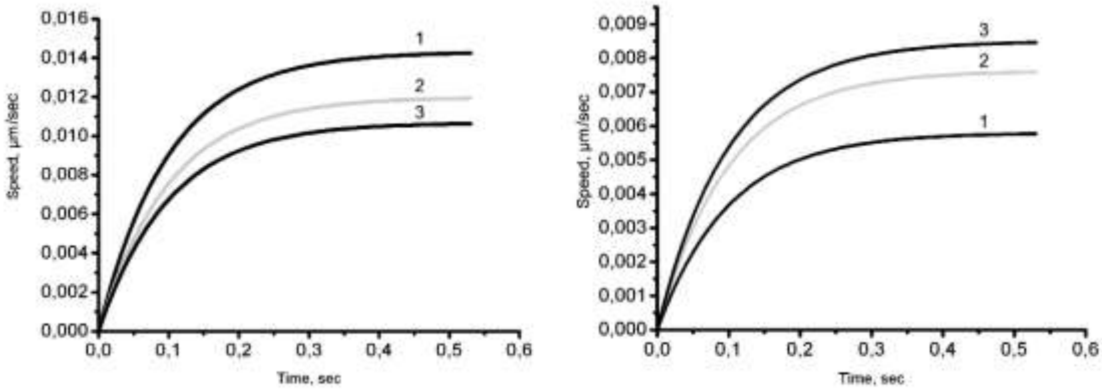


Figure 9. The change of the particle motion velocity with time obtained for different magnitudes of the degree of coherence of superposing waves in the case of particles moving:

a. along the peak of the field of time-averaged Poynting vector magnitude

b. along the minimum of the time-averaged Poynting vectors magnitude

Curves 1, 2, and 3 correspond to the degree of coherence, which equals 1, 0.5, and 0.25, respectively.

If the degree of mutual coherence of the superposed waves equals 0.2, the spatial distribution of the averaged Poynting vectors becomes more homogeneous, the modulation depth decreases considerably, and the velocities of nanoparticles become almost identical. When the degree of mutual coherence reaches 0.5, the relative velocities of the nanoparticles along the same trajectories are lower in comparison with velocities in case of complete mutual coherence of the superposed waves and lie in the vicinity of the average magnitudes for coherent and incoherent cases [49]. One observes the influence of mutual coherence of the superposed waves on motion velocities of nanoparticles with constant size and form in media with constant viscosity [49]. When analyzing the motion of test particles in the region of distributed magnitude of the Poynting vector, the influence of the parameters of superposing fields on the character of particle motion can be determined, cf. Fig. 9a, 9b.

It is clear [22, 48, 49], the degree of coherence of superposing waves in arrangement Fig. 7a determines the polarization structure of a field, *viz.* the spatial distribution of the Poynting vector. Under the same other conditions, *changing the degree of mutual coherence of superposing waves results in changing motion velocity of the test particles, what can serve as an estimating parameter for determining the coherence properties of superposing waves*. These differences in velocities of motion of nanoparticles are explained physically in the following manner: Increasing the share of incoherent part of the resulting field distribution causes a decrease of the modulation depth of the Poynting vector's spatial distribution, as well as decreasing resultant force magnitude along the lines of energy transfer, which induces the motion of nanoparticles. The increase of the degree of coherence brings about an accelerated particle motion in the field of averaged energy magnitudes.

4.2. Four-wave superposition of for changeable degree of mutual coherence of the components

For the case of superposition of four waves, see Fig. 10a, involving two sets of counter-propagating plane waves of equal intensities, linearly polarized in the plane of incidence and oriented at an angle of 90° with respect to each other, the spatial distribution of the time-averaged Poynting vectors with 2D periodicity is shown in Fig. 10b.

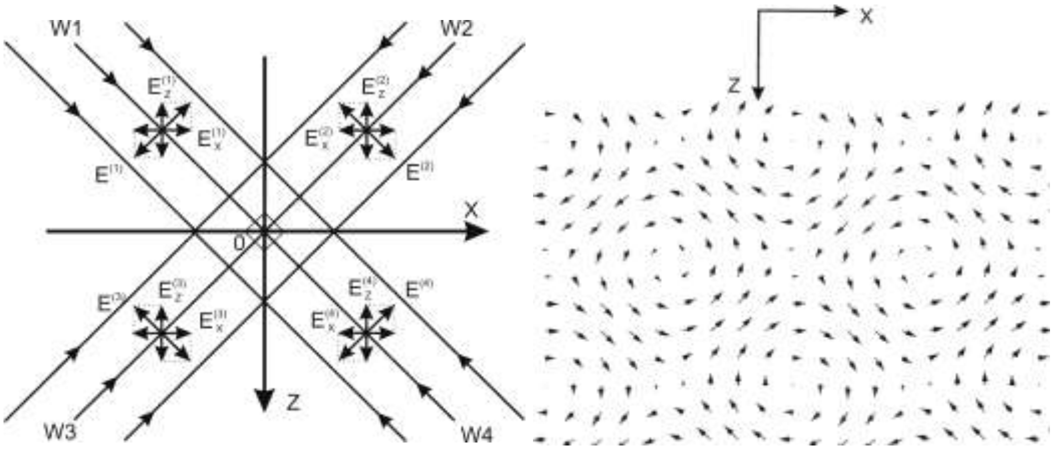


Figure 10.

a. Arrangement of superposition of four plane waves.

b. 2D distribution of the time-averaged Poynting vectors resulting from the superposition of four waves shown in Figure 10a.

As in the previous case, the lengths of the time-averaged Poynting vectors are proportional to their magnitudes. The nodal points in this distribution correspond to zero magnitudes (*singularities*) of the Poynting vector. In the following simulation, the diameters of the particles are changed to be comparable with a half-period of the corresponding spatial distribution of the Poynting vector. If the phase relations between four superposed beams are such that the modulation depth of the spatial distribution of the Poynting vector is maximal, the particle velocities will depend on the degree of mutual coherence between the interfering beams, see Fig. 11 a. In order to compare the influence of the temporal and spatial parameters of coherence on the motion of the nanoparticles, we have analyzed the maps of the time-averaged Poynting vector with a superposition of four plane waves over a large area. For that, we have tracked the nanoparticles' motion. The dependence of nanoparticles' velocities on the phase difference of the superposing beams has thus been revealed. So, in the case of pair-by-pair four opposite-in-phase superposed beams, particles become motionless. For that, the "opposite-in-phase" configuration covers the situation where two sets of mutually orthogonal standing waves are such that their nodes strictly coincide.

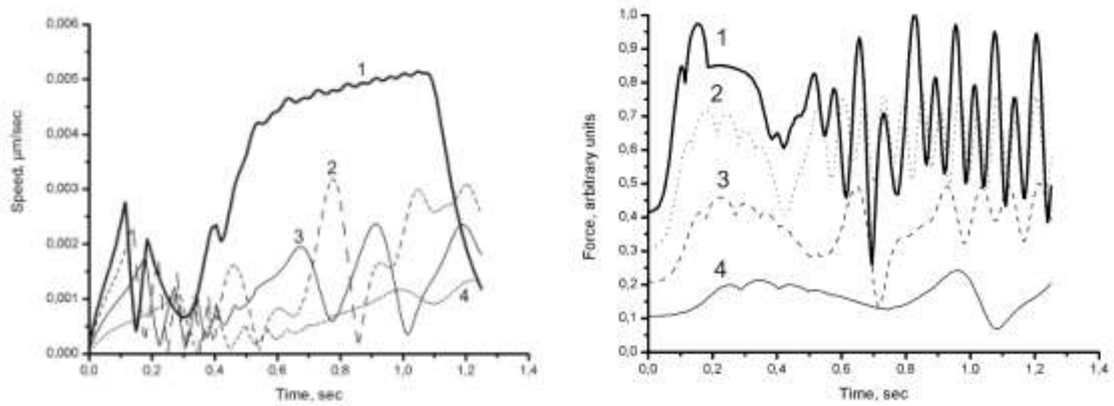


Figure 11.

a. The variation of motion velocity of a test particles in an averaged field of distributed Poynting vectors with the change of the degree of mutual coherence of the waves (four superposing waves are in phase): curve 1 – one of the waves is incoherent; curves 2, 3, 4 correspond to the degree of coherence 0.25, 0.5, and 0.75, respectively.

b. The change of the resultant force of the test particle motion in the time-averaged field of distributed Poynting vectors with the change of the degree of mutual coherence of the waves (four superposing waves are in phase): curve 1 – one of the waves is incoherent with all other waves; curves 2, 3 and 4 correspond to the degree of coherence 0.25, 0.5, and 0.75, respectively.

Increasing the degree of mutual coherence of the waves results in more uniform velocity magnitude of moving particles. The magnitude of the resultant force causing this motion under increasing degree of coherence, practically, does not change with time, see Fig. 11 b. The maximum depth of modulation for coherent equiphase waves determines the stable position of particles. *The chaotic state and the average particle velocity magnitude can be taken as a possible guideline in estimating the degree of coherence of superposing waves.*

The four-wave superposition of waves linearly polarized in the plane of incidence results in forming “cellular” structure of the resulting field distribution [11], which can be used for transfer (transporting) of the set of periodically positioned nanoparticles *as an entity* to desired zone. The use of strongly reflected test spherical particles provides obtaining more realistic notion on movement of particles in the field modulated in polarization in the incidence plane. So, the test particles are concentrated in zones (planes) of minima of the time-averaged Poynting vector and move along these planes. This situation reflects in the most adequate manner the processes of particle moving in the fields spatially modulated in polarization.

4.3. Experimental results

Direct experimental verification of the results of computer simulation is rather difficult. Spatial period of the polarization modulation resulting from superposition of plane waves meeting at right angle is less than a wavelength of the corresponding radiation. In this case, diagnostics of optical currents presumes the using test particles (preferably spherical) of size much less than the period of polarization distribution. That is why, direct visualization and diagnostics of such particle currents is hampered.

For verifying the results of above consideration, we have studied experimentally the influence of the field resulting from two-wave superposition (cf. Fig. 7a) with various combinations of their states of polarization on the test particles. In our experiments we have used spherical particles of hydrosol of gold with diameter 40 nm, approximately, for the period of spatial field distribution 449 nm. (Interested reader is persistently encouraged to find important practical details in [11].)

Periodical intensity distribution causes movement of particles and formation of the periodical distribution of particles’ concentration at the planes coinciding with interference minima of the intensity distribution at the area of superposition of two beams. These planes can be regarded as the analog of crystallographic planes in crystals. Direct visualization of particles and their currents is hampered due to small particle size. However, at planes of dense packing of particles self-diffraction takes place. We have observed this phenomenon for angles of meeting of two beams less than 40°. For right angle of meeting of the beams, the each self-diffracted beam propagates *along of* and *contrary to* the propagation direction of other of two superposing beams. Thus, it is impossible to discriminate the initial and self-diffracted beams. That is why, taking into account the Bragg law, we use, for diagnostics of periodical distributions of particles, the test laser beam with another wavelength, λ=532 nm. To form the same interference distribution (with the period 449 nm) with such wavelength the angle of meeting of two beams could be 72.6°. So, the angle of incidence of the probing beam must be 36.3° in respect to bisector of the writing beams. In this case, the Bragg law is fulfilled strictly for the probing beam. The mentioned angles are the angles of propagation in light-scattering media, in our case in water.

If two beams of red laser are polarized in the plane of incidence and the angle of meeting of them is equal to 90°, only polarization modulation takes place in the in the plane of observation. In this case the diffracted probing beam is present as well. The signal at the photodetector output is shown in Fig. 12 b. The diffracted probing beam is present, but is approximately of half the intensity in comparison with the case illustrated in Fig. 12 a. This experimental result is also in accordance with the result of computer simulation. The spatially modulated in polarization field is correlated with concentration of the test particles at the planes of minima of the time-averaged magnitude of the Poynting vector, and particles move along these planes. If two beams from red laser are linearly polarized, but one of them in the plane of incidence, while another one perpendicularly to this plane, the diffracted probing beam is absent, cf. Fig. 12 c. This shows that at the focal plane where the beams from red laser superpose, the periodical distributions of gold particles are absent. This experimental result is also in agreement with earlier computer simulation [43]. In other words, there are no any ordered optical currents being liable to optical diagnostics, as it has been made in previous case.

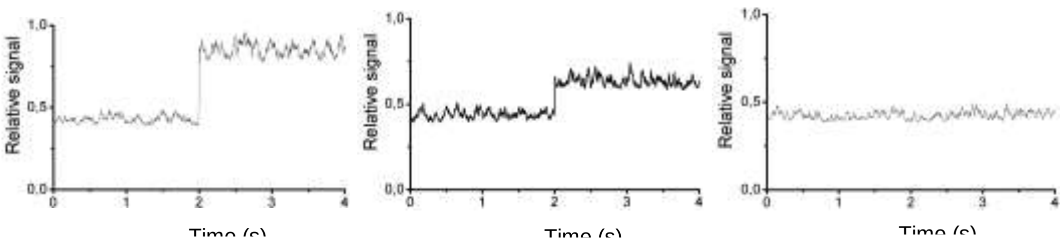


Figure 12. Relative signal of a photodetector (modulating plane-parallel plate is inserted on 2 sec at one interferometer leg and then removed) in the case when radiation of red laser is linearly polarized:

a. both beams are polarized in the plane perpendicular to the plane of incidence;

b. both beams are polarized in the plane of incidence;

c. one beam is polarized in the plane of incidence, while another one is polarized perpendicularly to this plane.

Thus, temporal and space peculiarities of particle's motion in optical fields without intensity modulation, but only due to polarization modulation causing the spatial modulation of the time-averaged Poynting vector (depending on the degree of mutual coherence of superpose waves) opens up new feasibilities for the use of such field characteristics and the parameters of nanoparticles motion for estimating the temporal coherence of the tested field. We have demonstrated a possibility of influence of only the polarization factor on formation of optical currents in liquids by the use of the principles of spatial polarization modulation in the observation plane. Besides, we have shown the possibility of diagnostics of optical currents using test particles of nanoscale. To all appearance, this metrology of fine structure of optical fields may be extended on polychromatic waves. The initial steps in this direction have been recently made [44-49].

5. Conclusions

Thus, new optical correlation approaches to metrology of partially coherent and partially polarized light fields have been developed. One of them reveals interconnection between polarization singularities inherent in partially polarized optical beams for the general case of partial mutual coherence of orthogonally polarized components. Another one concerns to exploring the spatial modulated time-averaged Poynting vector in completely and partially coherent paraxial light fields for control the motion of nanoparticles in optical currents. The represented approaches show fruitfulness of attracting the concepts and metrological tools of correlation optics in formation and investigation of unconventional polarization distributions that can be of usefulness in solving the problems of optical correlation diagnostics and optical telecommunications.

6. Acknowledgement

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