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INTENSITY WAVES AS OBSERVABLE PHENOMENA IN OPTICS

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Abstract. *By the interference of two coherent traveling waves of different frequencies, another traveling wave arises, at the level of the intensity of the field. In optics, the intensity wave, unlike the underground wave, is an observable phenomenon. The characteristics of the intensity waves are established and a photographic recording of the intensity wave obtained by the interference of two different frequency optical waves in a Mach-Zehnder arrangement is presented.*

Keywords: Coherence, interference, multifrequency optical fields

1. Introduction

By the interference of two coherent traveling waves, irrespective of the physical nature of the field, another wave structure arises at the level of the field intensity. Let us consider the simplest example, that of the interference of two coherent traveling waves of the same frequency. Figure 1a illustrates a snapshot of the interference field for two circular or spherical traveling waves of the same frequencies. A (circular or spherical) traveling wave, having the same frequency as that of the interfering waves, occurs, whose intensity is represented in Figure 1b. This intensity pattern can be interpreted as a *standing intensity wave*. In the following we shall see that *traveling intensity waves* can also be obtained, namely by the interference of two waves of different frequencies. For a clear distinction, we shall henceforth denominate the field wave resulting by the superposition of the two waves (Figure 1a) as *underground wave*.

In acoustics, for example, both the underground waves and the intensity waves are observable phenomena; the wave nature of the acoustical field is undoubted. It is not the same for the light.

Firstly, even if the light had a true wave nature, the traveling light waves couldn't be observed. They would travel with the speed of light, so a snapshot as that of Figure 1a couldn't be obtained. The frequencies of their oscillation would be of the order of $10^{14} \div 10^{15}$ Hz; no detection system placed at a point of a light field could detect such a frequency. For a light field, the underground waves are not observable phenomena. Their existence was merely inferred from the wave aspect of the intensity pattern, more precisely supposed on this basis.

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On the other hand, the feeble–light interference experiments [1] as well as the very recent single–photon experiments [2, 3] reconfirm the existence of the photon, which was called into question in the '70 by the semiclassical explanation of the photoelectric effect and of the Compton effect [4, 5]. The corpuscular aspect of the light, in the frame of the complementarity of our representation concerning the nature of light [6], is reenhanced.

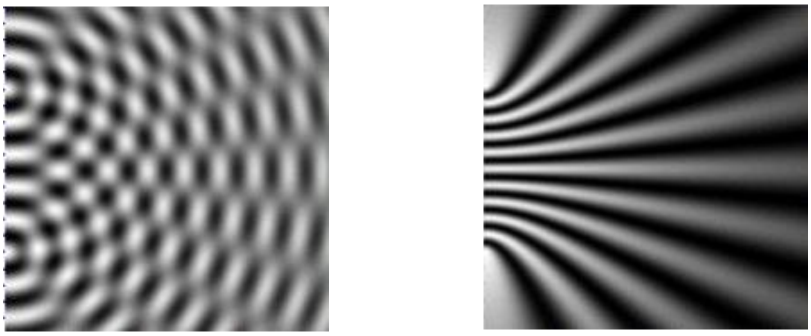


Figure 1. Snapshot of the interference field for two spherical waves of the same frequency
a. Traveling underground wave; **b.** Standing intensity wave.

Bearing in mind the above arguments, it is understandable why in a standard optical Young arrangement only the intensity pattern, like that of Figure 1b, is observable. Whether such an intensity distribution is built up by photons guided deterministically [7] or indetermininstically [8], this is still a matter of debate.

2. Interference of two waves of different frequencies and of different directions of propagation

Let us consider two uniform monochromatic plane waves of *different frequencies* (ω_1, ω_2) and *different directions of propagation* (the wave vectors $\mathbf{k}_1, \mathbf{k}_2$):

$$\begin{aligned}\Psi_1(\mathbf{r},t) &= a e^{i(\omega_1 t - \mathbf{k}_1\cdot\mathbf{r})}\\ \Psi_2(\mathbf{r},t) &= a e^{i(\omega_2 t - \mathbf{k}_2\cdot\mathbf{r})}\end{aligned} \tag{1}$$

For the sake of simplicity, we consider waves of equal amplitudes; but this doesn't introduce any essential restriction for our purposes.
The superposition field of these two waves:

$$\Psi(\mathbf{r},t) = \Psi_1(\mathbf{r},t) + \Psi_2(\mathbf{r},t) = 2a\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta\mathbf{k}}{2}\mathbf{r}\right)e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} \tag{2}$$

is a plane wave also, with the angular frequency :

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

and the wave vector :

$$\mathbf{k} = \frac{\mathbf{k}_1 + \mathbf{k}_2}{2} \tag{3}$$

But this wave is not a homogeneous one. Its amplitude changes both in space and time:

$$A(\mathbf{r}, t) = 2a \cos\left(\frac{\Delta\omega}{2} t - \frac{\Delta\mathbf{k}}{2} \cdot \mathbf{r}\right) \tag{4}$$

This last expression describes a uniform monochromatic plane wave. We shall denominate this wave *amplitude wave.* Its angular frequency and wave vector are respectively:

$$\Omega_A = \frac{|\Delta\omega|}{2} \ , \tag{5}$$

$$\mathbf{K}_A = \frac{\Delta\mathbf{k}}{2} \ , \tag{6}$$

where the difference $\Delta\mathbf{k}$ of the wave vectors $\mathbf{k}_2$ and $\mathbf{k}_1$ is taken in the order of indices which makes $\Delta\omega$ positive.

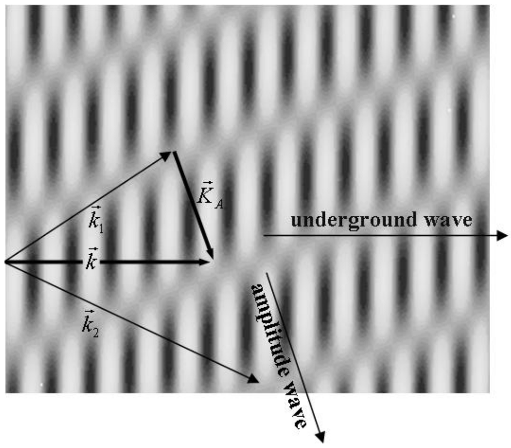


Figure 2. Double wave structure. Underground wave. Amplitude wave.

The field (for example an acoustical one) is that of a traveling wave $(\omega, \mathbf{k})$ modulated by a moving envelope — the amplitude wave $(|\Delta\omega|, \Delta\mathbf{k})$. The *underground wave,* $\cos(\omega t - \mathbf{k} \cdot \mathbf{r})$, and the amplitude wave (4) propagate in different directions: the first in the direction of the sum of the interfering waves' wave vectors, the second in direction of their difference (Figure 2). The velocity of the underground wave is:

$$u = \frac{\omega}{|\mathbf{k}|} = \frac{\omega_1 + \omega_2}{|\mathbf{k}_1 + \mathbf{k}_2|} \ . \tag{7}$$

This is the phase velocity of the field. The velocity of the amplitude wave is:

$$U_A = \frac{\Omega_A}{|\mathbf{K}_A|} = \frac{|\omega_2 - \omega_1|}{|\mathbf{k}_2 - \mathbf{k}_1|} \ . \tag{8}$$

For the wavelength of the amplitude wave we get:

$$\Lambda_A = \frac{2\pi}{|\mathbf{K}_A|} = \frac{4\pi}{|\mathbf{k}_2 - \mathbf{k}_1|} \ . \tag{9}$$

In Figure 2 is represented a snapshot pattern of the double wave structure.
The two waves propagate in different directions, indicated in the figure. The change in sign of the amplitude wave leads to a change in sign of the oscillation along the wavefront of the underground wave (the alternation of dark and light spots along the fronts of the underground wave).
In a slow quadratic detection (in optics, for instance, all detectors are of this kind), only the intensity envelope is recorded:

$$I(\mathbf{r},t) = 4a^2 \cos^2\left(\frac{\Delta\omega}{2}t - \frac{\Delta\mathbf{k}}{2}\cdot\mathbf{r}\right) = 2I_0[1+\cos(\Delta\omega t - \Delta\mathbf{k}\cdot\mathbf{r})] \ . \tag{10}$$

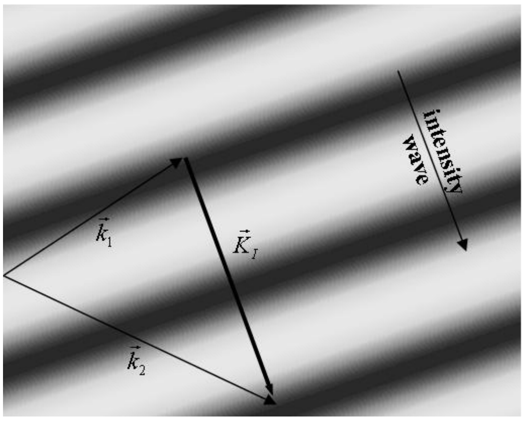


Figure 3. Intensity waves corresponding to a slow quadratic detection of the field of Figure 2.

All the information about the underground wave is lost. The distinction between the planes of positive and negative maxima of the amplitude is lost too, both being detected as maxima of the intensity. All that remains is the structure of moving interference fringes — the *intensity wave* (10) (Figure 3). The characteristics of this wave are:

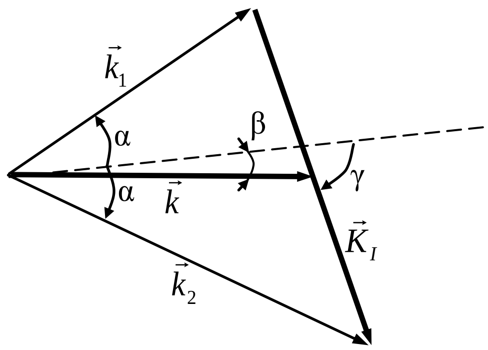
$$\Omega_I = |\Delta\omega| \ , \tag{11}$$

$$K_I = \Delta\mathbf{k} \ , \tag{12}$$

$$\Lambda_I = \frac{2\pi}{|\Delta\mathbf{k}|} \ . \tag{13}$$

The intensity wave is propagating with a velocity:

$$U_I = \frac{\Omega_I}{|\mathbf{K}_I|} = \frac{|\Delta\omega|}{|\Delta\mathbf{k}|} \ . \tag{14}$$



which is the group velocity of the field.

Figure 4. Wave vectors triangle

Let us put the above results in a more detailed form. In Figure 4 is drawn the wave vectors triangle of the interfering field. If the angles made by $\mathbf{k}_1$ and $\mathbf{k}_2$ with their bisector are $\pm\alpha$ then the planes of constant phase of the underground wave run in a direction which makes an angle

$$\beta = \text{arctg}\left(\frac{k_1 - k_2}{k_1 + k_2}\,\text{tg}\alpha\right) = \text{arctg}\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 + \lambda_2}\,\text{tg}\alpha\right) \tag{15}$$

with this bisector. The planes of constant amplitudes (and constant intensities) of the field travel at an angle γ to the bisector, given by:

$$\gamma = \text{arcctg}\left(\frac{\mathrm{k}_2 - k_1}{k_1 + k_2}\,\text{ctg}\alpha\right) = \text{arcctg}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\,\text{ctg}\alpha\right) \tag{16}$$

The characteristics of the underground wave are:

$$k = \frac{1}{2}|\mathbf{k}_1 + \mathbf{k}_2| = \frac{1}{2}\sqrt{k_1^2 + k_2^2 + 2k_1k_2\cos 2\alpha} \ , \tag{17}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2\cos 2\alpha}} \ , \tag{18}$$

$$u = \frac{\omega}{k} = \frac{\omega_1 + \omega_2}{\sqrt{k_1^2 + k_2^2 + 2k_1k_2\cos 2\alpha}} = \upsilon\frac{\lambda_1 + \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2\cos 2\alpha}} \ , \tag{19}$$

where υ is the velocity of a monochromatic wave in the given medium, supposed undispersive.

For the wavenumbers, wavelengths and velocities of the amplitude wave and of the intensity wave, from (6), (8), (9) and (12)–(14) is obtained:

$$K_I = 2K_A = \sqrt{k_1^2 + k_2^2 - 2k_1k_2\cos 2\alpha} \tag{20}$$

$$\Lambda_I = \frac{1}{2}\Lambda_A = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2\cos 2\alpha}} \tag{21}$$

$$U_I = U_A = \frac{|\omega_1 - \omega_2|}{\sqrt{k_1^2 + k_2^2 - 2k_1k_2\cos 2\alpha}} = \upsilon\frac{|\lambda_1 - \lambda_2|}{\sqrt{\lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2\cos 2\alpha}} . \tag{22}$$

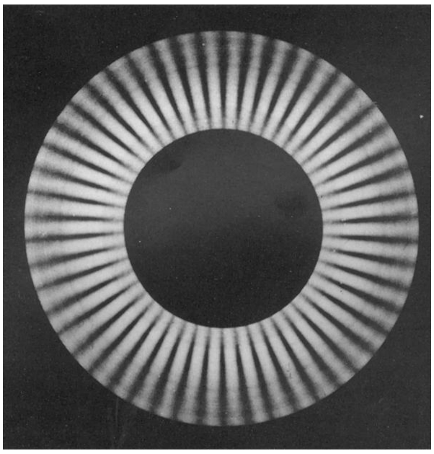


Figure 5. Photographic recording of an intensity wave

In Figure 5 we present a photographic recording of an intensity wave obtained by the interference of two different frequency optical waves in a Mach-Zehnder arrangement (Figure 6). He-Ne laser light was sent into the interferometer. In one of the arms, an optical frequency translator (F.T.) using two KDP crystal modulators in tandem was introduced. A shift of laser frequency of 600Hz, equal with the modulation frequency of the crystals was produced. The reference beam and the frequency-shifted beam, were mixed at the output of the interferometer. The output mirror was tilted so that the two interfering beams propagate in slightly different directions. A cylindrical lens, *L*, was used to expand the beams.

We have recorded the corresponding intensity wave by using the revolving disk method [9]: a moving disk, *D*, on which the photographic film is applied, is rotated by a motor, *M*, in a plane normal to the bisector of the directions of propagation of the interfering beams, which coincide with the optical axis of the arrangement (Figure 5). A small angle sectorial slit is placed in front of the disk, with its bisector along the interference fringes. Thus the film is impressed only as long as it moves parallel to the intensity wave, i.e. along $\Delta\mathbf{k}$. There exists a circumference on the disk, whose linear speed is equal to the velocity of the intensity wave (14). A ring of radial fringes is recorded along this circumference. Stroboscopic synchronism is achieved by means of the loop.

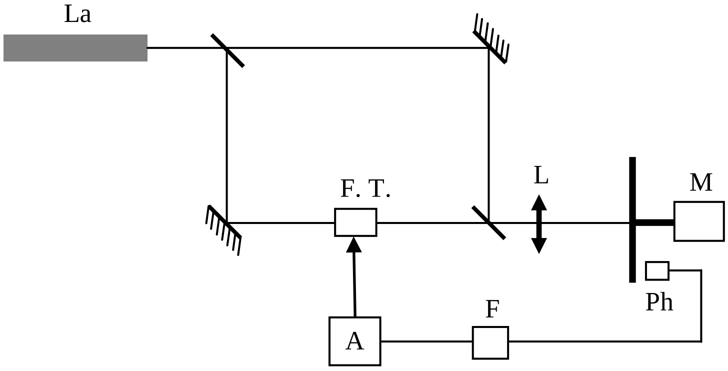


Figure 6. The experimental setup

Intensity waves in multifrequency optical fields produced by electrooptic modulation [10] in a Young arrangement were obtained too [11].

3. The generalized coherence

The intensity of the optical field is the simplest observable of the field. It is the first of a whole family of observables – the correlation functions of the field.

The standard Wolf's correlation function [12] does not succeed in describing the coherence of different frequency waves. Indeed, by taking the simplest case of two monochromatic waves of different angular frequencies, ω_1, ω_2 Wolf's coherence function:

$$\Gamma_{12}(\tau) = \left\langle V_1\left(t+\frac{\tau}{2}\right)V_2^*\left(t-\frac{\tau}{2}\right)\right\rangle \tag{23}$$

would not indicate their coherence:

$$\Gamma_{12}(\tau) = \left\langle a\,e^{i\omega_1\left(t+\frac{\tau}{2}\right)}a\,e^{-i\omega_2\left(t-\frac{\tau}{2}\right)}\right\rangle$$

$$= a^2 e^{i\frac{\omega_1+\omega_2}{2}\tau}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}e^{i(\omega_2-\omega_1)t}dt = 0 \tag{24}$$

whereas, in fact, the two waves are perfectly coherent.

A generalization of Wolf's coherence function was introduced [13], [14] for describing the coherence of different frequency optical fields too:

$$\Gamma_{12}(\sigma,\tau) = \left\langle V_1\left(t+\frac{\tau}{2}\right)V_2^*\left(t-\frac{\tau}{2}\right)e^{-i\sigma t}\right\rangle \tag{25}$$

where σ is a parameter in testing the different frequency (generally the multifrequency) coherence.

For the previous example, the *generalization coherence function* gives:

$$\Gamma_{12}(\sigma,\tau) = \left\langle a\,e^{i\omega_1\left(t+\frac{\tau}{2}\right)}a\,e^{-i\omega_2\left(t-\frac{\tau}{2}\right)}e^{-i\sigma t}\right\rangle$$

$$= a^2 e^{i\frac{\omega_1+\omega_2}{2}\tau}\left\langle e^{-i\left[\sigma-(\omega_1-\omega_2)\right]t}\right\rangle$$

$$= a^2 K \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} e^{-i[\sigma-\Delta\omega]t}dt = a^2 K \lim_{T\to\infty}\frac{\sin[\sigma-\Delta\omega]}{[\sigma-\Delta\omega]T}$$

$$= \begin{cases} 0 & for\ \sigma \neq \Delta\omega \\ a^2 K & for\ \sigma = \Delta\omega \end{cases} = a^2 e^{i\frac{\omega_1+\omega_2}{2}\tau}\delta(\sigma-\Delta\omega) \tag{26}$$

i.e., it indicates the existence of the coherence on the test frequency $\sigma = \omega_2 - \omega_1$.

The generalized coherence of multifrequency fields obtained by light modulation was analyzed this way [15].

Well-known, the standard Wolf correlation function has itself a wave propagation: its time evolution is governed by the standard wave equation too [12]:

We have established that the propagation of the generalized coherence is governed by the following equations:

$$\Delta_1\Gamma = \frac{1}{c^2}\left[\frac{\partial}{\partial\tau}+i\pi\sigma\right]^2\Gamma\,, \tag{27a}$$

$$\Delta_2\Gamma = \frac{1}{c^2}\left[\frac{\partial}{\partial\tau}-i\pi\sigma\right]^2\Gamma\,, \tag{27b}$$

which constitute a natural generalization of Wolf's equations and reduce to them for σ = 0.

The correspondence of the intensity waves in the field of optical polarization are the polarization waves — more precisely, the waves of states of optical polarization. Once again, the underlying waves are unobservable, the only observable phenomena being the polarization waves. Unlike the intensity waves, the polarization waves are indirectly observable phenomena. We have introduced the notion of *polarization waves*, we have given their theory [16] and we have revealed them experimentally [17].

Conclusions and perspective

The stationary intensity waves of the kind shown in Figure 1b were also obtained, generally in Young arrangements, for electrons [18], neutrons [19], atoms [20] and molecules [21]. A first issue of interest would be to verify that traveling intensity waves may also be obtained with all these material (non-zero rest mass) particles.
For some of these particles [18, 19], as well as for photons [2, 3], the stationary Young pattern was obtained in feeble fluxes, even in particle–by–particle regime. A second issue of interest would be to verify for photons and as well as for other particles that the traveling intensity structure persists in particle–by–particle regime too.

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