FORMATION AND STABILITY OF MULTIDIMENSIONAL LOCALIZED STRUCTURES IN OPTICS AND BOSE-EINSTEIN CONDENSATE: RECENT STUDIES

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Abstract. *We give a brief overview of recent results in the area of both* (2+1)- *and* (3+1)-*dimensional localized structures in some selected models in optics and Bose-Einstein condensate. We concentrate on the existence and robustness of these multidimensional localized structures and on the possibility of observation of* (3+1)-*dimensional solitons ("light bullets") in optical settings.*

Keywords: spatiotemporal optical solitons, light bullets, vortex (spinning) solitons, optical lattices, Bose-Einstein condensates

1. Introduction

In the past two decades there has been an increasing interest in the theoretical and experimental study of shape-preserving confined structures of light, which overcome either dispersion (temporal solitons), or diffraction (spatial solitons) [1]-[4]. These temporal and spatial solitons are special cases of a larger class of nonlinear phenomena in which both temporal and spatial effects are coupled and occur simultaneously. The space-time coupling occurring when a pulsed optical beam propagates through a nonlinear medium leads to unique nonlinear effects, such as the spatiotemporal collapse in the case of anomalous group-velocity dispersion (GVD), pulse splitting if the GVD of the medium is normal, the formation of fully confined (in both transverse spatial dimensions) light pulses, i.e., the creation of spatiotemporal optical solitons [1], etc. The multidimensional localized structures have attracted a great deal of attention both in optics and in the field of atomic Bose-Einstein condensate (BEC).

In optics, the localized multidimensional structures are spatially confined on the order of wavelength. They represent the "particle-like" counterpart of the more common extended light structures. The optical media that might sustain such self-guiding structures should be nonlinear, i.e., their refractive index should depend on the light intensity. Different kinds of nonlinearities of optical materials such as absorptive, dispersive, second-order (quadratic), third-order (Kerr-like) can be used to prevent temporal dispersion/spatial diffraction of light beams or both of

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them. The field of temporal/spatial optical solitons emerged from these fundamental studies of interaction of intense laser beams with matter. This research area is now in a mature stage; temporal optical solitons are currently created in monomode optical fibers and have led to a mature technology in nonlinear optics and photonics, whereas spatial optical solitons are currently created in various experimental conditions in laboratory and are now awaiting technological implementation in all-optical processing of information.

However, there exist a third kind of optical solitons, the so-called *spatiotemporal optical solitons*, alias “light bullets” [5, 6], which are spatially confined pulses of light, i.e., electromagnetic wave packets self-trapped in both space and time. The term “light bullet” arises because the spatiotemporal optical soliton can be thought of as a tiny bead of light propagating long distances without changing its shape. These localized physical objects could be used as information carriers in future all-optical processing information systems. It is believed that the “light bullets” are the ideal information units in both serial and parallel transmission and processing information systems.

The solitons in media with the cubic self-focusing nonlinearity, obeying the nonlinear Schroedinger (NLS) equation, are unstable in two and three dimensions, because of the occurrence of beam collapse [7, 8]. However, several possibilities to arrest the intrinsic wave collapse were considered, such as the use of quadratically nonlinear optical media that support solitons for all physical dimensions [9, 10] and where (2+1)-dimensional light bullet formation was achieved by generating the necessary anomalous GVD via achromatic phase matching [11], the use of saturable [12, 13] and nonlocal [14, 15] optical media, materials with competing nonlinearities [16, 17], the propagation of (3+1)-dimensional localized structures in confining two- or three-dimensional optical lattices [18-22], the formation of multidimensional fundamental and vortex (spinning) dissipative solitons in media with gain and loss described by the cubic-quintic Ginzburg-Landau equation [23, 24], and the propagation of *discrete light bullets* in one- and two-dimensional photonic lattices [25-27].

The landmark experimental work [11] reporting the formation of a (2+1)-dimensional spatiotemporal optical soliton used a very clever scheme to control the GVD along one spatial axis. The beam self-trapping occurred only along one spatial transverse dimension of a two-dimensional optical beam. It is well known that by reflecting a beam from a diffraction grating, the nonspecular orders have their energy wavefront tilted relative to their phase velocity wavefront, with different spectral components having different tilts; pulse compression in time based on this principle was achieved by using the cascaded nonlinearity in second-order nonlinear optical materials, such as lithium iodate and beta-barium borate (BBO) [11].

Quadratic spatiotemporal solitons in the cascaded limit with highly elliptically shaped beams were generated by using the above mentioned second-harmonic generation crystals. Along the long axis of the optical beam cross-section, the diffraction length was longer than the length of the crystal so that no beam diffraction occurred. However, along the short beam axis, the diffraction length was about one fifth of the crystal length and it is along this transverse coordinate that the beam behaved like a spatial optical soliton. The pulse width of about 100 fs was used in this experiment, with the grating-engineered GVD, to match the dispersion length to the diffraction length in order to form a spatiotemporal optical soliton ("light bullet"). It was demonstrated that along the short beam axis no spreading occurred both in space and in time, a characteristic feature of a (2+1)-dimensional light bullet. Thus for propagation over five characteristics lengths, the beam size (pulse duration) was about 50 microns (100 fs). It is worthy to mention that in this experiment it was also reported the formation and the propagation over several dispersion lengths of temporal solitons in quadratic nonlinear optical media.

This work is organized as follows. In Section 2 we briefly overview the studies of existence, stability and robustness of three-dimensional vortex solitons in both conservative and dissipative settings. The problem of formation of stable three-dimensional light bullets in lower dimensional photonic lattices is discussed in Section 3. Section 4 is devoted to the study of a few approaches to get stable two- and three-dimensional vortices in Bose-Einstein condensates. Finally, Section 5 concludes the paper.

2. Stable vortex (spinning) solitons in three dimensions

The *localized optical vortices* (alias vortex solitons), have drawn much attention as objects of fundamental interest, and also due to their potential applications to all-optical information processing, as well as to the guiding and trapping of atoms. In the core of an optical vortex the complex electromagnetic field is equal to zero, however the circulation C of the gradient of the phase of the complex field on an arbitrary closed contour around the vortex core is a multiple of 2π, i.e., $C=2\pi S$, where the integer S is the *topological number* of the vortex ("spin"). Thus the phase dislocations carried by the wavefront of a light beam are associated with a zero-intensity point (a vortex core); the phase is twisted around such points where the light intensity vanishes, creating an optical vortex.

It is worthy to mention that unique properties are also featured by vortex clusters, such as rotation similar to the vortex motion in superfluids. The complex dynamics of two- and three-dimensional soliton clusters in optical media with competing nonlinearities has been studied too [28, 29]. Various complex patterns based on both fundamental (nonspinning) solitons and vortices were theoretically investigated in optics and in the usual BEC models governed by the Gross-Pitaevskii equation with both local [30] and nonlocal nonlinearity [31].

Stable nondissipative spatiotemporal spinning solitons (vortex tori) with the topological charge S=1, described by the three-dimensional NLS equation with focusing cubic and defocusing quintic nonlinearities were found to exist for sufficiently large energies [16]. This result also holds for the case of competing quadratic and self-defocusing cubic nonlinearities [17]. A general conclusion of these studies is that stable spinning solitons are possible as a result of competition between focusing and defocusing optical nonlinearities. We have also performed a comprehensive stability analysis of three-dimensional dissipative solitons with intrinsic vorticity S governed by the complex Ginzburg-Landau equation with cubic and quintic terms in its dissipative and conservative parts [23, 24]. It was found that a necessary stability condition for all vortex solitons, but not for the fundamental ones (S=0), is the presence of nonzero diffusivity in the transverse plane. The fundamental solitons are stable in all cases when they exist, while the vortex solitons are stable only in a part of their existence domain. However, the spectral filtering (i.e., the temporal-domain diffusivity) is not necessary for the stability of any species of dissipative solitons. Stability domains were found for (3+1)-dimensional vortex solitons (vortex tori) with "spin" S=1, 2, and 3, suggesting that spinning solitons with any vorticity S can be stable in certain portions of their existence domains [24]. Typical examples of stable (3+1)-dimensional solitons with vorticities S=0, 1, and 2, which form in dissipative cubic-quintic media are shown in Fig. 1.

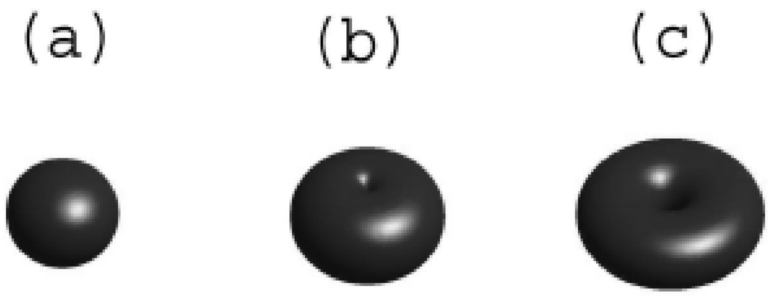

Figure 1. Isosurface plots of optical intensity: **a.** S = 0; **b.** S = 1; **c.** S = 2.

It is worthy to mention that the signature of an optical vortex with topological charge S can be detected by looking at the unique structure of the interference pattern of the vortex field with a plane wave. In Fig. 2 we show the interferograms corresponding to S = +1 and S = -1 [Fig. 2 (a)-Fig. 2(b)] and to S = +2 and S = -2 [Fig. 2 (c)-Fig. 2(d)]. The typical "fork-like" dislocations in the vortex core are clearly visible in the panels of Figure 2.

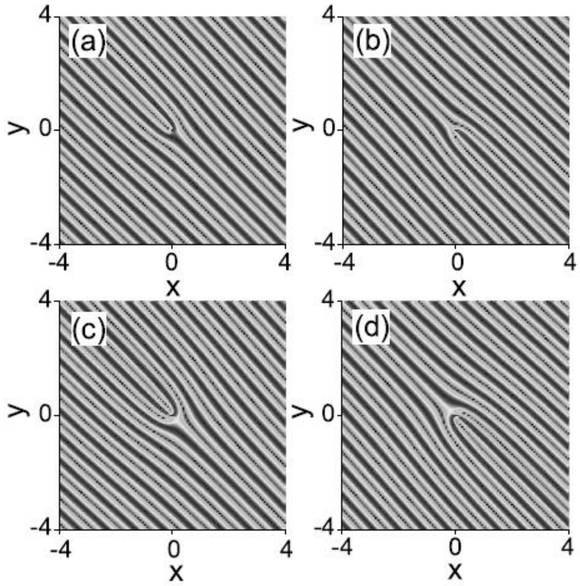

Figure 2. Vortex interferogram:
a. S = +1; **b.** S = –1; **c.** S = +2; **d.** S = –2.

Figure 3 shows a typical example of the robustness of non-dissipative light bullets with vorticity S=1 forming in cubic-quintic nonlinear media; recall that the S=1 vortex tori is stable if its energy is larger than a certain threshold [16]. It is clearly seen in Fig. 3 that the stable vortex is able to absorb the white noise perturbation and to clean up itself.

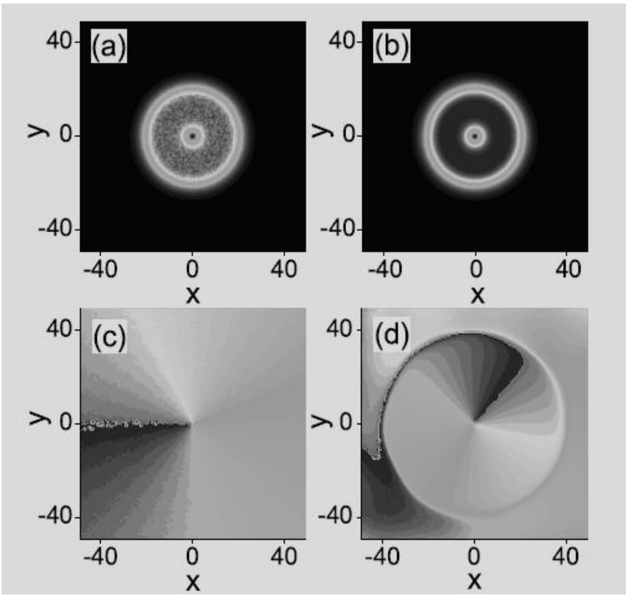

Figure 3. Self-cleaning of a randomly perturbed stable vortex with S=1:
a. Input intensity; **b.** Output intensity; **c.** Input phase; **d.** Output phase.

The spinning soliton can be easily generated from an input Gaussian field with a nested vortex (see Fig. 4 for a typical situation). The input Gaussian optical field [see Fig. 4(a)] with a phase dislocation at the vortex core [see Fig. 4 (c)] evolves towards a *stable flat-top like vortex soliton*, with “spin” S=1, see Figs. 4(b) and 4(d).

3. Stable three-dimensional light bullets in two-dimensional photonic lattices

A very promising way to arrest the collapse in cubic (Kerr-type) focusing media is to use two-dimensional nonlinear photonic lattices in a three-dimensional environment [18-20]. The existence and stability of three-dimensional spatiotemporal solitons in self-focusing cubic Kerr-type optical media with an imprinted two-dimensional harmonic transverse modulation of the refractive index was studied in detail in Ref. 19. It was demonstrated that two-dimensional photonic Kerr-type nonlinear lattices can support stable one-parameter families of three-dimensional spatiotemporal solitons provided that their energy is within a certain interval and the strength p of the lattice potential, which is proportional to the refractive index modulation depth, is above a certain threshold value.

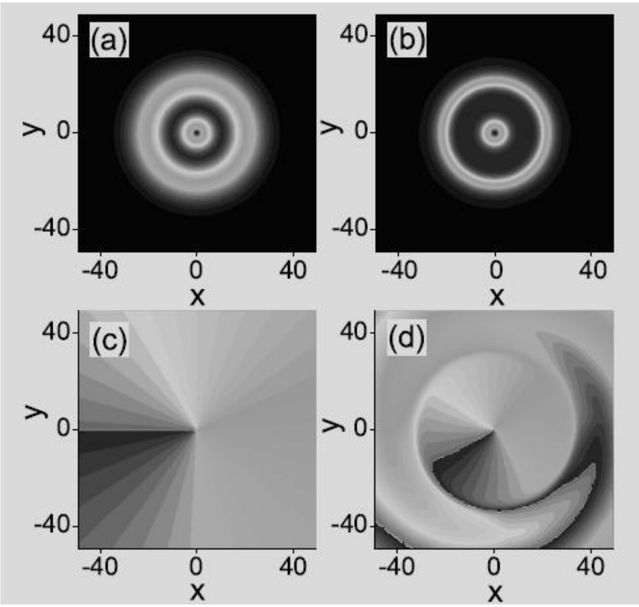

Figure 4. Formation of a stable vortex soliton with S=1 from a Gaussian field: **a.** Gaussian input; **b.** Output vortex with S=1; **c.** Input phase; **d.** Output phase.

As a consequence of the imprinted two-dimensional photonic lattice, the nonlinear localized states exist only for nonlinear wave numbers (propagation constants) larger than some minimum values (the edge of the band gap). The minimum propagation constant increases with the increase of the lattice strength parameter; recall that for the NLS equation the minimum propagation constant is equal to zero. Families of three-dimensional spatiotemporal solitons in two-dimensional harmonic lattices exist whenever their energy exceeds a certain minimum value and are linearly stable in the intermediate-energy regime and for sufficiently high lattice strengths. Remarkably, for sufficiently large values of the lattice strength parameter p, the Hamiltonian-versus-energy (soliton norm) curves plotted in Fig. 5 display two cusps, instead of a single one as in other 2D and 3D nondissipative (Hamiltonian) nonlinear dynamical systems. This unique two-cusp structure of the soliton norm-Hamiltonian diagram is the so-called *"swallowtail"* catastrophe and is quite rare in physics [19, 20]. Remarkably, this unique swallowtail bifurcation occurs also in the study of stability of three-dimensional solitons with vorticity S=1 supported by a two-dimensional harmonic lattice if the lattice strength is large enough [22].

Recently we have introduced *discrete surface light bullets* forming in both one-dimensional [25] and two-dimensional [26] photonic lattices. We analyzed spatiotemporal light localization near the edge of semi-infinite arrays of weakly coupled nonlinear optical waveguides or in the corners and the edges of two-dimensional photonic lattices and demonstrated the existence and stability (in certain regions of their existence domain) of continuous-discrete spatiotemporal surface solitons, the so-called discrete surface light bullets [25, 26]. We have shown that their properties, such as power (energy) thresholds for their formation are strongly affected by the presence of the photonic lattice truncation. Recently we analyzed the interactions between continuous-discrete spatiotemporal optical solitons and we observed a variety of collision scenarios and different outcomes, such as soliton fusion, symmetric and asymmetric scattering [27].

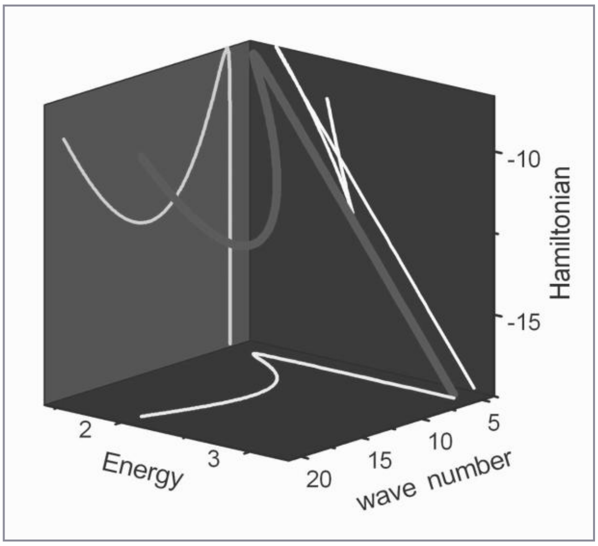

Figure 5. Typical energy (soliton norm)-wave number-Hamiltonian diagram for 3D light bullets confined by 2D optical lattices. Here the lattice strength parameter is p=20.

4. Stable two- and three-dimensional solitons and vortices in attractive Bose-Einstein condensates

The creation of multidimensional solitons and vortices built of matter waves is a great challenge to the experiment. The current situation in this field was summarized in two recent reviews [3, 5]. It is well known that the nonlinear Gross-Pitaevskii equation adequately describes the BEC dynamics in terms of the mean-field single-atom wave function [3].

In Ref. 32 we performed an accurate investigation of stability of localized vortices in an effectively 2D "pancake-shaped" trapped BEC with negative scattering length (self-attractive condensate). The states with vorticity S=1 were found to be stable in a third of their existence region. For the isotropic 3D configuration, the stability interval expands to about 65% of the existence domain. However, all vortices with S=2 were found to be unstable. This study was extended to the case of 3D self-attractive Bose-Einstein condensate trapped in anisotropic parabolic potentials, with arbitrary aspect ratio Ω between trapping lengths in the transverse plane (x,y) and along the third coordinate z [33]. The relative size of the stability domain for 3D vortices with S=1 increases with the decrease of the aspect ratio in terms of the soliton norm N, but decreases in terms of the chemical potential μ. As in the 2D case, all vortex tori with S≥2 were found to be unstable, while the stability of the fundamental (S=0) solitons obeys the standard Vakhitov-Kolokolov criterion, i.e., the states for which we get positive slopes of the soliton energy (E)-soliton norm (N) curves correspond to stable S=0 solutions.

The existence and stability of solitons in Bose-Einstein condensates with attractive interatomic interactions, described by the Gross-Pitaevskii equation with a full (three-dimensional) periodic confining potential, were investigated in a systematic form in Ref. 21. We found a one-parameter family of stable 3D solitons in a certain interval of values of their norm N, which is related to the number of atoms, provided that the strength of the optical lattice potential exceeds a threshold value. The minimum number of ^{7}Li atoms in the stable solitons is about 60, and the energy of the soliton at the stability threshold is about six recoil energies in the lattice. The respective energy (E) versus soliton norm (N) diagram features two cuspidal points, resulting in a typical *swallowtail-like pattern* (see Fig. 6), which is a generic feature of 3D solitons supported by both quasi-two-dimensional [19] or three-dimensional harmonic lattice potentials [21]. Remarkably, this unique swallowtail bifurcation occurs also in the study of the stability of 3D solitons with vorticity S=1 supported by a 2D harmonic lattice if the lattice strength is large enough [22]. Notice also that a repulsive BEC confined in a 3D optical lattice supports spatially localized (in all three dimensions) vortex structures which are remarkably robust and which possess highly nontrivial particle flows [34].

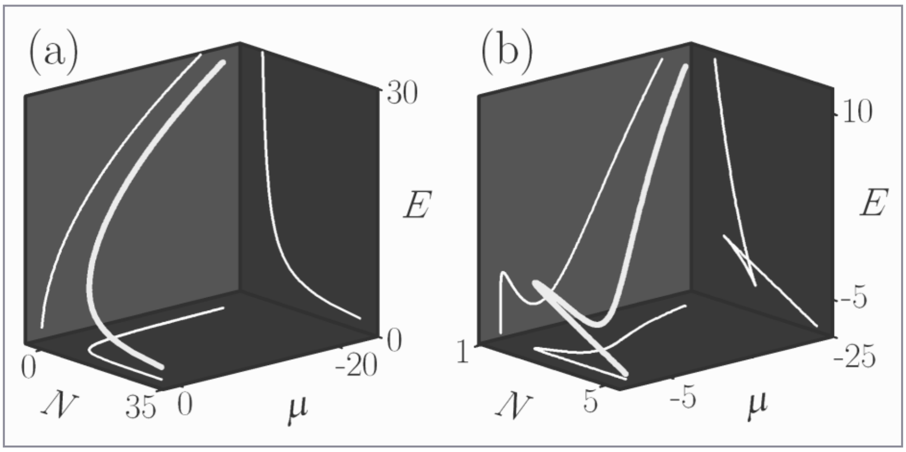

Figure 6. Typical soliton norm-chemical potential-energy diagram for 3D solitons in attractive Bose-Einstein condensates loaded in 3D optical lattices.
a. Lattice strength p=0; **b.** Lattice strength p=3.

Conclusions

As concerning the possible practical implementation of the light bullet concept we mention here two realistic physical settings.

Firstly, the conditions for low-power spatiotemporal soliton formation in arrays of evanescently-coupled silicon-on-insulator (SOI) photonic nanowires have been thoroughly analyzed recently [35]. It was shown that pronounced soliton effects can be observed even in the presence of realistic loss, two-photon absorption, and higher-order GVD. The well established SOI technology offers an exciting opportunity in the area of spatiotemporal optical solitons because a strong anomalous GVD can be achieved with nanoscaled transverse dimensions and moreover, the enhanced nonlinear response resulting from this tight transverse spatial confinement of the electromagnetic field leads to soliton peak powers of only a few watts for 100-fs pulse widths (the corresponding energy being only a few hundreds fJ). The arrays of SOI photonic nanowires seem to be suitable for the observation of discrete surface light bullets because a suitable design of nanowires can provide dispersion lengths in the range of 1 mm and coupling lengths of a few millimeters (for 100-fs pulse durations) [35].

Secondly, a potential approach to form stable 3D light bullets might be based on the concept of engineered structures composed of different optical materials featuring either strong nonlinearity or strong suitable GVD but not necessarily both together at a given wavelength [36].

The implementation of such idea along the propagation (longitudinal) direction showed that light bullet formation is possible for significantly large tandem domains in the case of quadratic spatiotemporal solitons [36].

Very recently, it was shown that stable 3D light bullets do form in transverse radially periodic metamaterial structures consisting of alternating rings made of highly dispersive linear materials and rings made of strongly nonlinear media (with cubic saturable optical nonlinearities) [37].

We conclude with the hope that this brief overview on recent exciting developments in the area of multidimensional localized structures in optics and Bose-Einstein condensate will perhaps inspire further investigations.

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