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# PERFORMANCES OF SERVOCONTROL MECHANISMS UNDER THE INFLUENCE OF EXTERNAL PARAMETERS AT VOLUMETRIC PUMPS WITH AXIAL PISTONS

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**Rezumat.** *Servomecanismele hidraulice şi pneumatice utilizează elemente de execuţie hidraulice, respectiv pneumatice. Ele sunt larg utilizate în practică pentru amplificarea forţei sau momentului, având ca principală caracteristică proporţionalitatea dintre mărimea de intrare (poziţie sau tensiune) şi mărimea de ieşire (poziţie, forţă sau moment) în regim staţionar. Performanţele care interesează utilizatorul sunt în general următoarele: • precizia statică; • precizia la viteză constantă; • răspunsul la semnal treaptă; • constanta de timp; • curba de răspuns în frecvenţă; • rigiditatea la ieşire, similară cu sensibilitatea la forţele exterioare. Servocomanda pompelor cu pistoane axiale poate fi influenţată de anumiţi factori externi, cum ar fi: elasticităţile legăturilor dintre servocomandă şi structura de bază, elasticitatea dintre cilindri şi sarcină sau elasticitatea mecanismelor de comandă. În analiza prezentată este descrisă o metodologie de calcul a parametrilor şi valoarea lor limită pentru a evita apariţia erorilor în funcţionarea pompelor cu pistoane axiale.*

**Cuvinte cheie:** precizie, rigiditate, sensibilitate, elasticitate.

**Abstract.** *Hydraulic or pneumatic servomechanisms use hydraulic and pneumatic actuators. They are widely used in practice to increase force or torque, with the main feature represented by the proportionality between the input parameter (position or voltage) and the output parameter (position, force or torque) under stationary regime. The performances in which the user is interested are generally the following: • Static accuracy; • Steady speed accuracy; • The response to step signal, the time constant; • Frequency response curve; • Output stiffness similar to the sensitivity to external forces. The servo control of axial piston pumps can be influenced by external factors, such as: the elasticity of the connections between servo control and the basic structure, elasticity between cylinders and load or elasticity of control mechanisms. The analysis presented describes a methodology for calculating the parameters and their limit value to avoid errors in the operation of axial piston pumps.*

**Keywords***:* accuracy, stiffness, sensitivity, elasticity.

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## 1. Introduction

This article is established to a detailed study of the dynamic behaviour of hydraulic servomechanisms installed in ideal conditions. The main objective of the study is the theoretical determining of the quantitative influence of constructive parameters on accuracy and stability. The principle of an elementary electric servomechanism is shown in Figure 1:

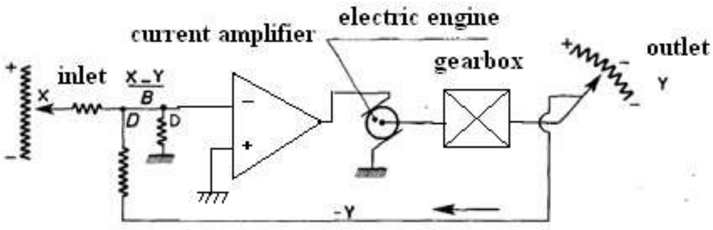


**Fig. 1.** The principle of an electric servo elementary.

This servomechanism is simpler than a hydraulic servomechanism, this study being useful for electro-hydraulic servomechanisms [1]. Is reproduced at output a moving equal to moving from input. For this, are materialized the input and output by X and Y voltages obtained at the variables potentiometers of input and output. The resistance mounting of Figure 1 provides in point D, to a reduced scale, the voltage X-Y. This is amplified in the electronic amplifier which commands an electric motor whose speed is proportional to the received voltage.

$$\frac{dY}{dt} = K(X - Y) \tag{1}$$

K is a constant which takes into account the gear and amplifier.

This equation becomes

$$X = Y + \frac{1}{K}\frac{dY}{dt} \tag{2}$$

where in symbolic writing,

$$\frac{X}{Y} = 1 + \frac{p}{K} \tag{3}$$

*p* is a polynomial of degree I. In conclusion the constant "K" leads to an improved accuracy and reduces the time constant.

But equation (2) is incomplete because it does not take into account the masses inertia in series with the power circuit of the engine. The mechanism comprising also in composition inertia masses is shown in Figure 2.

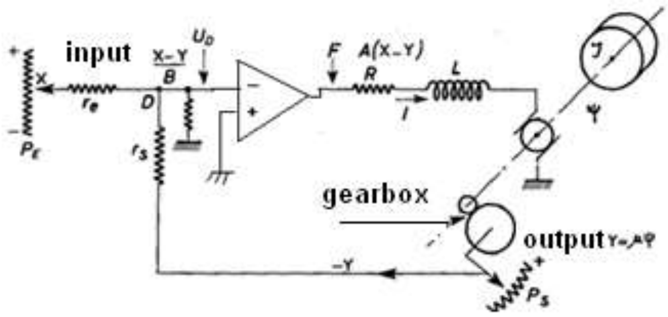


**Fig.2**. The mechanism with inertial masses.

Where ψ is the rotation angle in radians of the engine shaft at output, *J* is the total reduced inertia torque to the engine shaft; X and Y are the voltages obtained at the terminals of the potentiometers $P_E$ and *Ps*; by *I* is denoted the inertia of the engine circuit; *R* is the sum of the resistances of the engine and amplifier's resistance; *L* is the total inductance of the circuit.

In point "D" appears voltage (we consider $r_e = r_s$).

In point "F" appears a tension *A(X-Y)*

The engine torque is:

$$C = \lambda I \tag{4}$$

λ is the engine constant.

The counter-electromotive force of the engine is:

$$E_c = \lambda \frac{d\Psi}{dt} \tag{5}$$

The engine power is:

$$E_c I = C \frac{d\Psi}{dt} \ or \ \frac{C}{I} = \frac{E_c}{\left(\frac{d\Psi}{dt}\right)} = \lambda \tag{6}$$

which inserted into the servomechanism equation we obtain:

$$J \frac{d^2\Psi}{dt^2} = \lambda I \tag{7}$$

(probably for overcoming inertia is used the whole engine torque).

$$\begin{aligned} &A(X - Y) = \lambda \frac{d\Psi}{dt} = RI + L\frac{dI}{dt} \\ &Y = \mu\psi \end{aligned} \tag{8}$$

μ is a coefficient which takes into account the voltage applied to the potentiometer.

By eliminating the parameters ψ and I in equations (6), (7) and (8), we obtain:

$$\begin{aligned} &I = \frac{J}{\lambda} p^2\psi = \frac{J}{\lambda\mu} p^2 Y \\ &A(X - Y) = \frac{\lambda}{\mu} pY + \frac{RJ}{\lambda\mu} p^2 Y + \frac{LJ}{\lambda\mu} p^3 Y \\ &\frac{X}{Y} = 1 + \frac{\lambda}{\mu A} p + \frac{RJ}{\lambda\mu A} p^2 + \frac{LJ}{\lambda\mu A} p^3 \end{aligned} \tag{9}$$

By these two examples it is shown that the ratio between the input / output is in the polynomial form in the variable "p". In this form the equation (9) is useful for the study of stability and the servomechanism's performances.

**Stability Servomechanism Conditions.**

Equation (9) is a function of the form

$$\frac{X}{Y} = 1 + ap + bp^2 + cp^3 + \ldots \tag{10}$$

For more complex servomechanisms, higher three order terms appear.

This function is "the reverse transfer function" of the servomechanism and Y/X is the transfer function of the servomechanism. In harmonic pulse regime $x = a\omega - c\omega^3$ it is equivalent to:

$$\frac{X}{Y} = 1 + ai\omega - b\omega^2 - ci\omega^3 + \ldots \tag{11}$$

X and Y representing here the complex amplitudes of the input and output quantities of the sinusoidal regime. This function is plotted in the complex plane shown in Figure 3.

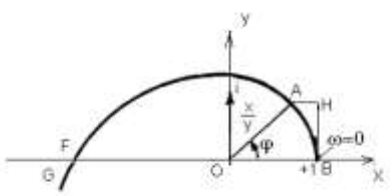


**Fig. 3.** Representation function X / Y in the complex plane.

with $x = 1 - b\omega^3;\ y = a\omega - c\omega^3$ (11, a, b)

For ω = 0, it has a value of + 1.

For very small ω the term, $ai\omega$ the tangent in point B is vertically, the term in ω provides

$$AH = 1 - b\omega^2$$

all the A points on the curve corresponding to a determined pulse ω.

It is also given the ratio $\left|\frac{X}{Y}\right| = OA$, the phase difference between the entry X and exit Y being given by angle $\langle xOA = \varphi \bullet X$ being in advance against to Y, which corresponds to a delay at the output as against the input.

## 2. Static Accuracy

The static accuracy is theoretically infinite; it is rendered in equation (10):

$$\frac{X}{Y} = 1 + ap + bp^2 + cp^3 + \dots \tag{12}$$

which transcribed into ordinary differential equation becomes:

$$X = Y + a\frac{dY}{dt} + b\frac{d^2Y}{dt^2} + C\frac{d^3Y}{dt^3} \tag{13}$$

The difference is δ

$$\delta = X - Y = a\frac{dY}{dt} + b\frac{d^2Y}{dt^2} + C\frac{d^3Y}{dt^3} \tag{14}$$

Statistically all second-order terms are zero. The difference δ ≠ 0. In reality the friction and the gaps are to be a dead zone which can decrease by a precise execution by increasing the amplification compatible with stability, this can be achieved using a performing distribution element (proportional distributor or servo valve).

## 3. Accuracy at Constant Speed

At constant speed the equation (13) reduces to

$$\delta = X - Y = a\frac{dY}{dt} \tag{15}$$

The difference δ is proportional to the speed, the coefficient of proportionality is homogeneous in time; it is often called *the time constant of the servomechanism*.

So that the speed variation be small, “a” must be as small as possible.

## 4. Accuracy in Harmonic Regime

X and Y representing the complex amplitude of the input quantities and pulsation ω see Figure 4 [1].

$\delta = X - Y$ , the complex amplitude error φ

The relative error $\frac{\delta}{Y}$ corresponds to $\frac{\delta}{Y} = \frac{X}{Y} - 1$ ; it is shown in Fig. 4 by vector + 1 • A.

If we replace in equation (10) $p=i\omega$, it becomes:

$$\frac{\delta}{Y} = \frac{X - Y}{Y} = ai\omega - b\omega^2 - ci\omega^3 \tag{16}$$

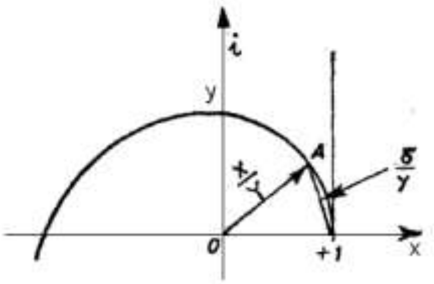


**Fig. 4.** The accuracy representation in harmonic regime.

This error is very small because the coefficients a, b, c are reduced, while the rest are compatible with the stability.

## 5. Frequency Response Curve

Essential nonlinearity from the mathematical model of a servomechanism is included in the distributor feature. The numerical simulations and the systematic experiments conducted on the mechanohydraulic servomechanisms dynamics led to the conclusion that if the effect of friction is minimized through structural and constructive measures and the technological errors are negligible their dynamic behaviour can be linear considered in a wide range of frequencies. To illustrate this observation, Figure 5 shows a typical response to a sinusoidal signal with amplitude comparable to the maximum opening of the distributor, its windows being rectangular, and the cover being zero. Bode's diagram Figure 6 a, b indicates that the servomechanism can fall into the category of the "comfortable" because at the resonance we have about - 12 dB attenuation. This confirms also the a periodic aspect of the response at step signal. Design or manufacturing errors, and technological restrictions can lead to a nonlinear behaviour of the servomechanisms. Their most important specific nonlinearities are the positive covering, the limiting of the distributor stroke and the gaps in the command chain [4].

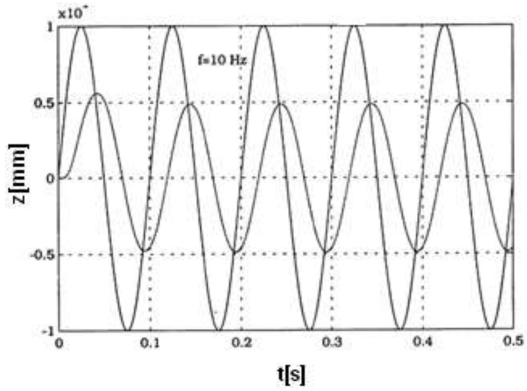


**Fig. 5.** The frequency response of a typical servomechanism.

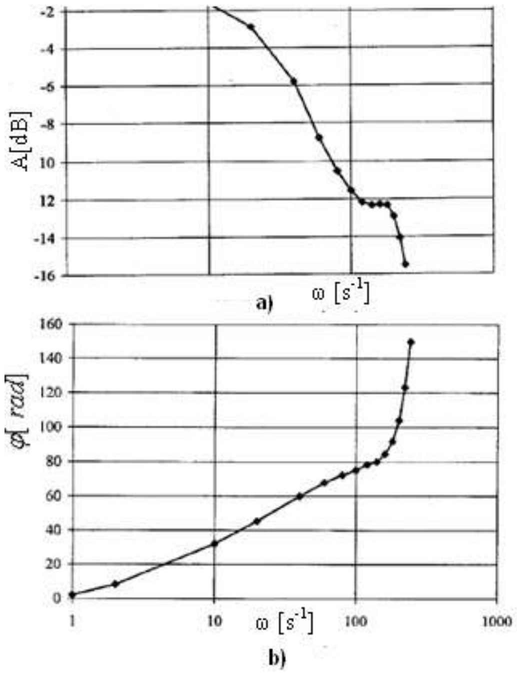


**Fig. 6.** Bode's diagram
a) Attenuation – pulsation diagram, b) Phase pulsation diagram

## 6. The Output Stiffness Similar to the Sensitivity of the External Forces

Sometimes the below calculations do not accurately reflect some effective behaviours of the servo command.

It must involve certain neglected previously parameters. These parameters are essentially connecting and fixing different elasticities:

- the fixing servocommand elasticity at the strength structure;
- the elasticity connection between the cylinder and the load;
- the elasticity of the actuator.

We present below to quantify the importance of these parameters and the level at which they should be considered (or, in the case of a project, the imposed limits to prevent or minimize the disruption in operation).

**6.1. The fixing elasticity of servocommand and the connection elasticity between cylinder and load [2]**

6.1.1. Equation presentation

Notations (see Figure.7)

z difference of the loading position of the load;

z', the position of the movable part of the cylinder (z' ≠ z if the stiffness connection is not infinite);

z the "position of the fixed cylinder considered part (and are not strictly fixed in the case in which are not connection parts with infinite stiffness);

N, the stiffness connection between cylinder and load;

F, the stiffness fixing.

(We shall write the transfer function neglecting the friction).

The flow through the distributor is:

$$Q_A = K_e = K(y - z') = \frac{K}{\lambda}(x - z')$$

The cylinder chambers volume: $\begin{cases} V_A = V_t + S(z' - z'') \\ V_B = V_t - S(z' - z'') \end{cases}$

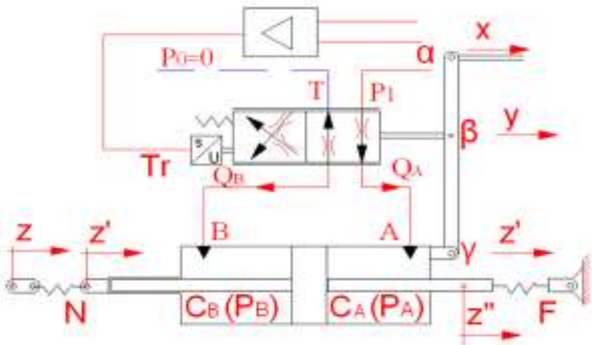


**Fig. 7.** The fixing elasticity of the servocommand and the connection elasticity between cylinder and load.

The flow equation will be:

$$\frac{K}{\lambda}(x - z') = (z' - z'')pS + \frac{V_t}{2B}p(P_A - P_B)$$

which can be written as:

$$\frac{K}{\lambda}(x - z) = Spz + \frac{V_t}{2B}p(P_A - P_B) + (z' - z)\left(\frac{K}{\lambda} + pS\right) - z''pS$$

Where:
$\lambda = \alpha\gamma / \beta\gamma$ - Report multiplication of the lever, S - the usable cylinder area.
The equation of balance for load:

$$N(z' - z) = z(r + mp^2)$$

The equation of balance for the fixed part of the cylinder:

$$(P_A - P_B)S = -Fz''$$

The equation of balance for the mobile part of the cylinder: $(P_A - P_B)S = N(z' - z)$
Eliminating the

$$z' - z = \frac{z}{N}(r + mp^2); \quad (P_A - P_B) = \frac{N}{S}(z' - z) = \frac{z}{S}(r + mp^2)$$

and on

$$z'' = -\frac{S}{F}(P_A - P_B) = -\frac{z}{F}(r + mp^2)$$

We obtain immediately the required transfer function:

$$H_1 = \frac{z}{x - z} = \frac{K / \lambda S}{\frac{K}{\lambda S}\frac{r}{N} + p\left[1 + \frac{V_t r}{2BS^2} + \frac{r}{N} + \frac{r}{F} + \frac{K}{\lambda S}\frac{m}{N}p + \left(\frac{V_t m}{2BS^2} + \frac{m}{N} + \frac{m}{F}\right)p^2\right]}$$

(17)
Whether by introducing two different sizes defined above (the hydraulic stiffness and the inverse of the time constant) $r_h = 2BS^2 / V_t \ et\ \omega_f = K / \lambda S)$ :

$$H_1 = \frac{z}{x - z} = \frac{\omega_f}{\omega_f \frac{r}{N} + p\left[1 + \frac{r}{r_h} + \frac{r}{N} + \frac{r}{F} + \omega_f \frac{m}{N}p + \left(\frac{m}{r_h} + \frac{m}{N} + \frac{m}{F}\right)p^2\right]} \quad (18)$$

As we saw above the ratio r / $r_h$ is negligible in relation to 1.
On the other hand, the rigidities N and F, if defects are attached to the servocommand are always higher than r.
Therefore, practically, the ratio r / N and R / F can neglect in relation to 1; it results the final form of the transfer function (1):

$$H_1 = \frac{z}{x - z} = \frac{\omega_f}{\omega_f \frac{r}{N} + p\left[1 + \omega_f \frac{m}{N}p + \left(\frac{m}{r_h} + \frac{m}{N} + \frac{m}{F}\right)p^2\right]} \quad (19)$$

## Conclusions

The above calculation is made for a servocommand in which the detection of the back size (the distributor body position) is made mandatory at the cylinder level (z'). In an electro-hydraulic servocommand, for example, the detector (potentiometer) can be placed either on the cylinder or on the load. The question is which are the advantages of the two solutions.

**Detector on cylinder** – the same commissioning in equation for a mechanical servocommand; the same results**.**

**Detector on load** - the flow of the distributor is equal to:

$$Q_A = K_e = K(y - z) = \frac{K}{\lambda}(x - z),$$

therefore the flow equation is:

$$\frac{K}{\lambda}(x - z) = (z' - z'')pS + \frac{V_t}{2B}\, p\,(P_A - P_B)$$

written as

$$\frac{K}{\lambda}(x - z) = S\, p\, z + \frac{V_t}{2B}\, p\,(P_A - P_B) + (z' - z)\, p\, S - z''\, p\, S\,,$$

Finally we come with the same approximation as above:

$$H_1 = \frac{z}{x - z} = \frac{\omega_f}{p\left[1 + \left(\frac{m}{r_h} + \frac{m}{F} + \frac{m}{N}\right)p^2\right]} \tag{20}$$

When installing the detector directly on the load:

- It eliminates the position error;
- It eliminates the amortization term;
- It stores the term in p3 responsible for the reduction of $\omega_c$.

## R E F E R E N C E S


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