Conference Proceedings of the Academy of Romanian Scientists
Online Edition PRODUCTICA Scientific Session, 27th 28th may 2016
ISSN 2067-9564 Volume **8**, Number **1**/**2016**

ANALYTICAL METHOD TO EVALUATE THE MAINTENANCE PARAMETERS OF A PRODUCTION LINE USING MARKOV CHAINS

Elena – Iuliana GINGU (BOTEANU)[1], Miron ZAPCIU[2]

Rezumat. *În ultimul timp, Managementul Ciclului de Viaţă al Produsului (PLM) a atras atenţia nu numai cercetătorilor cât şi producătorilor. În această lucrare, una dintre etapele Managementului Ciclului de Viaţă al Produsului, este analizată. Etapa de Mentenanţă a Managementului Ciclului de Viaţă al Produsului este aplicată în cadrul acestei cercetări, cu scopul de a estima MTBF (Timpul Mediu de Bună Funcţionare) şi MTTR (Timpul Mediu de Reparaţie). Aceste valori pot fi folosite pentru a îmbunătăţi eficienţa proceselor din companie. Un studiu de caz real este propus pentru a fi analizat cu ajutorul lanţurilor Markov. În cele din urmă, mai mulţi algoritmi aplicaţi pe un post de lucru cu două maşini sunt dezvoltaţi. Folosind aceşti algoritmi şi metoda descompunerii, parametrii maşinilor (rata de producţie, MTBF, MTTR) pot fi estimaţi pentru orice linie în flux.*

Abstract. *Lately, Product Lifecycle Management (PLM) has attracted not only researchers' attention but also the manufacturers'. In this paper, one of the phases of Product Lifecycle Management is analysed. The Maintenance phase of Product Lifecycle Management is applied to this research in order to estimate the MTBF (Mean Time between Failures) and MTTR (Mean Time to Repair). These values can be used to improve the efficiency of the processes in the company. A real case study is proposed to be analysed with the help of Markov chains. Finally, several algorithms applied on a workstation with two machines are developed. Using these algorithms and decomposition method, the machine parameters (production rate, MTBF and MTTR) can be estimated for any flow line.*

Keywords: Product Lifecycle Management, production, maintenance, Markov chains.

1. Introduction

Product Lifecycle Management (PLM) is an information management system that can incorporate data, processes and people in a developed company.

PLM represents the management of all phases of a product's lifecycle, from the emergence of the new idea to commercialization and decline.

[1] PhD Student, Faculty of Engineering and Management of Technological Systems, POLITEHNICA University of Bucharest (e-mail: iuliana_boteanu@yahoo.com);
[2] Professor, Faculty of Engineering and Management of Technological Systems, POLITEHNICA University of Bucharest; Academy of Romanian Scientists, Splaiul Independenţei 54, 050094, Bucharest, Romania (e-mail: miron.zapciu@upb.ro).

PLM helps a company to capitalize its industrial *know-how* by standardizing each stage of a product cycle and integrating specific constraints related to the sector of the respective activity.

Fig. 1. PLM phases [9]

In this research, Markov chains and decomposition method in the Maintenance phase of Product Lifecycle Management (PLM) will be included. In the phase of Maintenance, the analytical model using Markov chains helps companies to estimate the MTBF (Mean Time between Failures) and MTTR (Mean Time to Repair) of the line in order to improve the efficiency of the processes in the company.

2. Theoretical Framework

2.1. Some notions of reliability theory

We consider a device which is subject to failures. The reliability function $R(t)$ is defined as follows [1-4, 8]:

$$R(t) = P[\text{There is no failure on } [0, t] \,/\, \text{The system works at } t = 0] \tag{1}$$

This is clearly a non-increasing function of time. From this we can define the Mean Time to Failure (MTTF) as:

$$MTTF = \int_0^\infty R(t)dt \tag{2}$$

When the system can be repaired, the maintainability function is:

$$M(t) = 1\text{-}P[\text{There is no repair on } [0, t] \,/\, \text{The system is down at } t = 0] \tag{3}$$

The Mean Time to Repair is then defined as:

$$MTTR = \int_0^\infty (1 - M(t))dt \tag{4}$$

We can also define the availability function:

$$A(t) = P[\text{The system works at } t] \tag{5}$$

For a reparable system we have a succession of up times and of down (and then repair) times. In this case, the average value of availability that we denote by *A* is:

$$A = \frac{MTTF}{MTTF + MT\,\mathrm{TR}} \tag{6}$$

Another parameter useful in reliability theory is the failure rate, defined as follows:

$$\lambda(t) = \frac{1}{dt}\frac{R(t) - R(t + dt)}{R(t)} \tag{7}$$

It can be seen that the failure rate is constant when the Time to Failure is exponentially distributed [1-4, 8].

2.2. Modelling and performance evaluation of production systems using Markov chains

Markov chains were rapidly recognized for their important influence on representation and their ability to model a wide range of real life problems as well as for the quality of performance indices they give with a relatively small computing effort. One of the first significant applications was the work of Erlang on the modelling of telephone traffic [3]. Markov chains are also well suited for reliability computations, even for complex systems [4]. Similarly they can be used for modelling and performance evaluation of manufacturing systems when they reveal some random behaviour (breakdowns, random time to machine a part) [8].

Markov chains are the natural basis for most of the results in queuing theory, and as such they have now a wide range of applications in the field of communication and computer systems [8].

Modelling using Markov chains:

- The first and generally the most difficult part resides in the modelling process itself. We must first identify the Markov states. Our advice to do this is to list the physical stable situations of the system and for each of them ask the following question: can this situation be represented by a state? In other words, does the future only depend on the fact that we are in this situation? If the answer is clearly *no*, the situation must be decomposed in several states.
- Identify the *Up* states and the *Down* states.
- If the system is reparable, the Markov chain is usually ergodic and the stationary performance, and especially the availability and the mean production rate of the system can be computed using the stationary probability distribution.
- If the system is not reparable (which rarely happens in production contexts), after the identification of the up states, one can compute the *MTTF* using transient calculations.
- The most common situation is the one when the systems are reparable. The corresponding Markov chain is ergodic but it is of interest to compute the *MTTF* and the *MTTR* of the system.

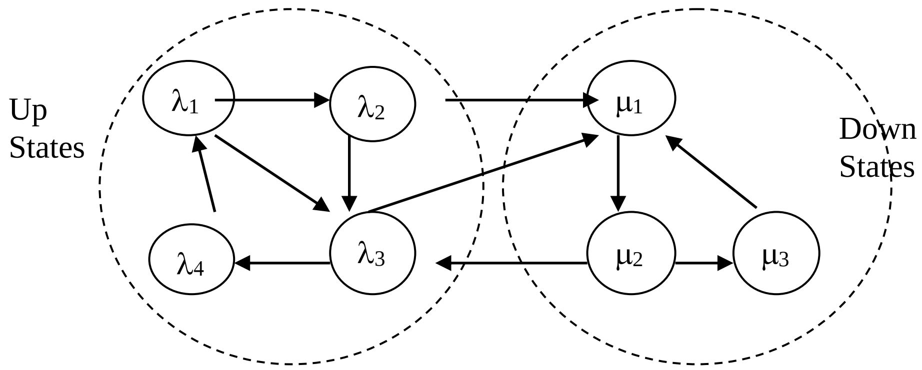


Fig. 2. The Markov chain of a reparable system.

Relative to this decomposition of states, the Markov generator can be decomposed as:

$$\Lambda = \begin{bmatrix} Q & R \\ T & S \end{bmatrix} \tag{8}$$

If we are interested in the Time to Failure, it means that we just deal with what happens while we are passing through the up states before the first failure. This corresponds to the transient behaviour of a Markov chain in which the down states are ergodic, which means that we ignore the repairs [1,2,5-7]. All the facts described previously can be very well expressed by the Markov generator:

$$\tilde{\Lambda} = \begin{bmatrix} Q & R \\ 0 & S \end{bmatrix} \tag{9}$$

The *MTTF* is the average time spent in the transient states, which can be computed using the usual transient calculations [1,5].

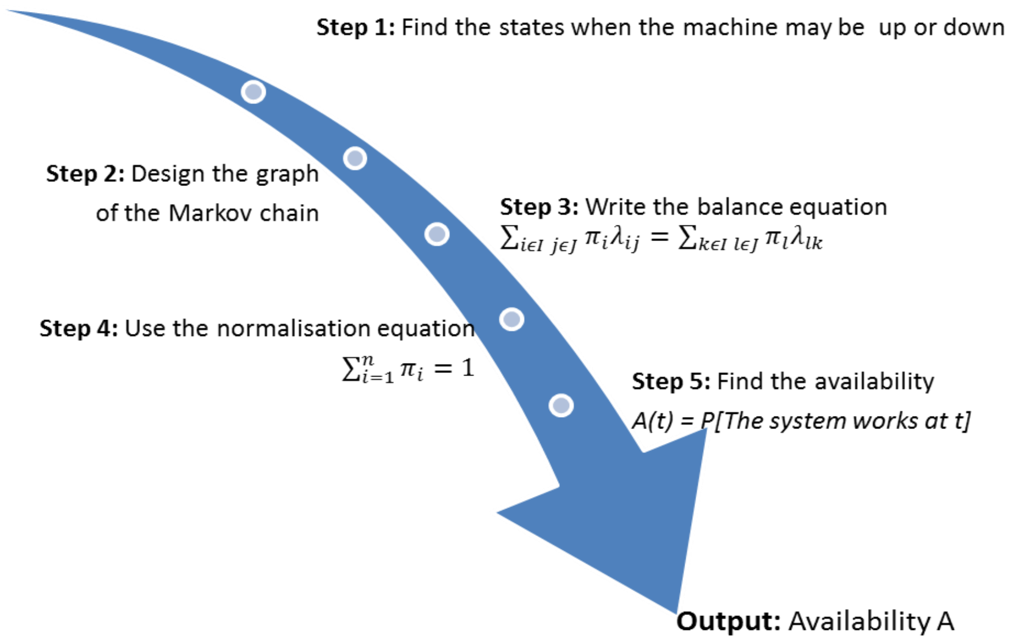
The *MTTR* can be computed in the same way.

3. Research Methodology

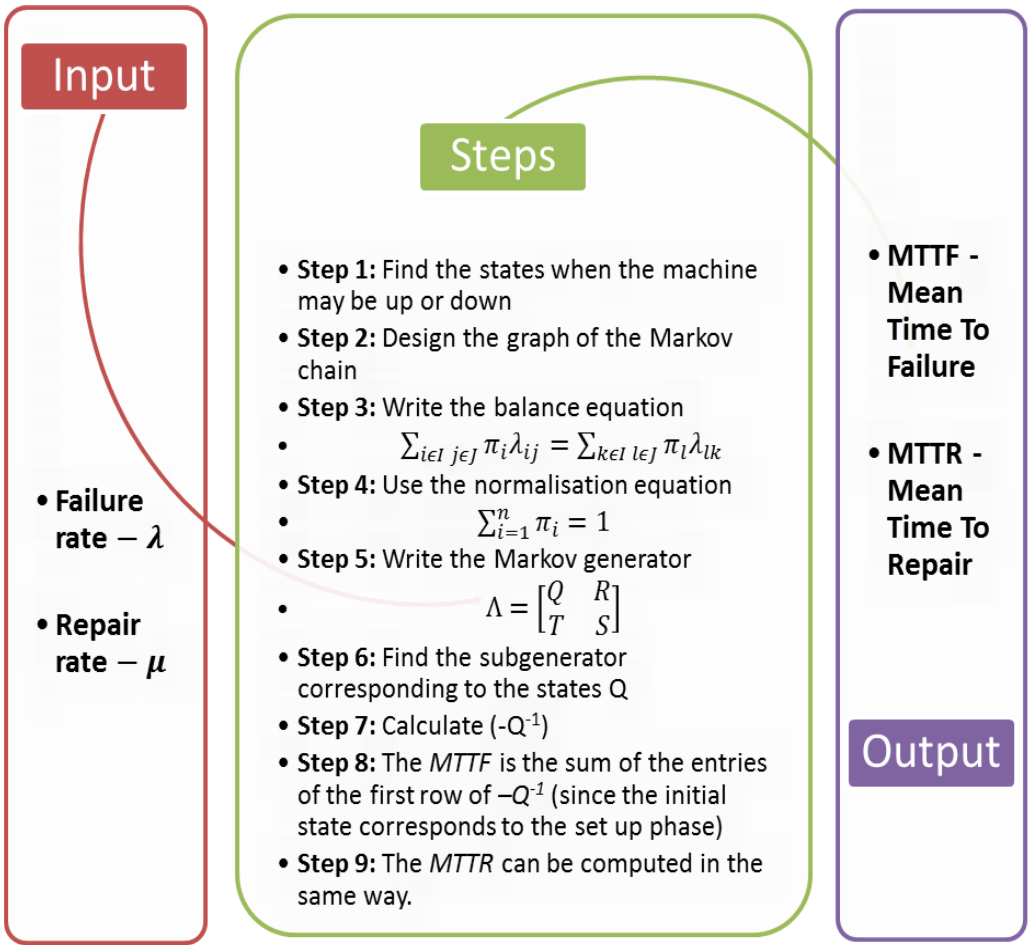
In order to estimate the MTBF (Mean Time between Failures), MTTR (Mean Time to Repair) and the availability of the system, we propose the following algorithms:

Algorithm 1. Availability of the system

Input: failure rate λ and repair rate μ



Algorithm 2. Estimation of the MTTF and MTTR



4. Numerical Results

We consider the following line:

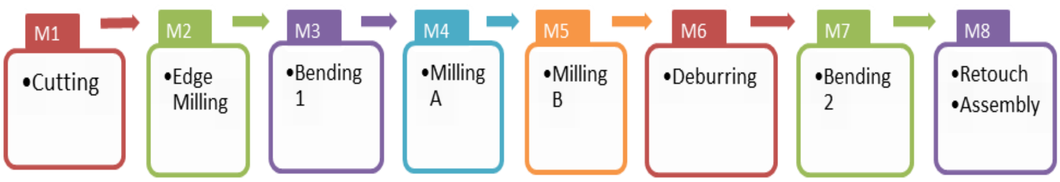


Fig. 3. Flow line with 8 machines – Headrest support

Then we isolate only two machines and afterwards we try to find the Availability, MTTF, MTTR, and MTBF.

To estimate the availability and the MTBF and MTTR, we consider the following assumptions and results:

- M_1 when it is working, it fails with the rate λ_1=0.005, M_2 when it is working, it fails with the rate λ_2 =0.008.
- When it is down, if the maintenance operator is working on it, M_1 is repaired with the rate μ_1 =0.09. When it is down, if the maintenance operator is working on it, M_2 is repaired with the rate μ_2=0.11.
- M_1 has always priority in the following sense: if both machines work properly, then M_1 works as well; if both machines are down, then the maintenance operator works on M_1, even if he has already begun to repair M_2.

Step 1

There are four states when both machines may be up or down:

- x_1 : M_1 is working, M_2 is working, but it is idle,
- x_2 : M_2 is working, M_1 is down and under repair,
- x_3 : M_1 and M_2 are down and M_1 is under repair,
- x_4 : M_1 is working, M_2 is down and under repair.

Step 2

The graph of the Markov chain is the following:

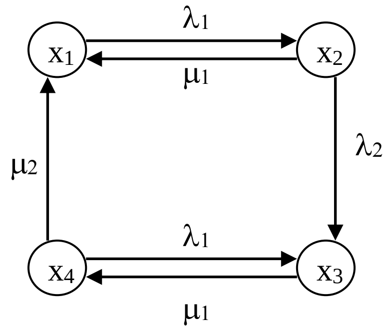


Fig. 4. Markov chain graph

Some remarks on the model:

- there are no direct transitions between x_1 and x_3 or x_2 and x_4, because two independent events will happen in the same *dt* with a probability which is in $(dt)^2$; then they are "neglected" before first order transitions;
- in state x_4, M_2 is under repair, if M_1 fails we go to x_3, we abandon the repair of M_2, but when we go back to x_4 and to the repair of M_2, all the previous repair time is lost because of the no memory property.

Step 3

The balance equations may be chosen as:

$$\begin{aligned} \pi_1.\lambda_1 &= \pi_2.\mu_1 + \pi_4.\mu_2, \\ \pi_3.\mu_1 &= \pi_2.\lambda_2 + \pi_4.\lambda_1 \\ \pi_2.\lambda_2 &= \pi_4.\mu_2. \end{aligned} \tag{10}$$

Step 4

Using the normalization equation:

$$\pi_1+\pi_2+\pi_3+\pi_4=1, \quad (11)$$

we finally get:

$$\begin{array}{ll} \pi_1 = 0.9435 & \pi_2 = 0.0485 \\ \pi_3 = 0.0045 & \pi_4 = 0.0035 \end{array} \quad (12)$$

The availability of the system is $(1 - \pi_3) = 0.9955$.

Steps 5-6

To compute the *MTTF* we start from the state x_1. We consider the down state x_3 as absorbing and we consider the average time spent in the transient states which will be x_1, x_2 and x_3, in our case.

The sub generator corresponding to these states is:

$$Q = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 \\ \mu_1 & -(\mu_1+\lambda_2) & 0 \\ \mu_2 & 0 & -(\mu_2+\lambda_1) \end{bmatrix} \quad (13)$$

$$Q = \begin{bmatrix} -0.005 & 0.005 & 0 \\ 0.09 & -0.098 & 0 \\ 0.11 & 0 & -0.115 \end{bmatrix} \quad (14)$$

We could inverse the Q matrix and sum the entries of the first row of $(-Q^{-1})$, to get the *MTTF*, but we can save some efforts by noting that Q is block triangular and the entry (1, 3) of $(-Q^{-1})$ is certainly null. This observation is comforted by the fact that a sojourn in x_4 can only occur after a stay in x_3 and then cannot participate in the MTTF (starting from x_1).

Then, we have just to reverse the following matrix:

$$Q = \begin{bmatrix} -0.005 & 0.005 \\ 0.09 & -0.098 \end{bmatrix} \quad (15)$$

Step 7

And we get these numerical values:

$$-Q^{-1} = \begin{bmatrix} 2450 & 125 \\ 2250 & 125 \end{bmatrix} \quad (16)$$

Step 8

Therefore, the MTTF = 2575.

Step 9
In the same way we can calculate the MTTR.

$$-Q^{-1} = \begin{bmatrix} 10.2 & 0 \\ 0 & 8.7 \end{bmatrix} \tag{17}$$

MTTR= 10.2.
The availability calculated with this method is:

$$A = \frac{MTTF}{MTTF + MTTR} = \frac{2575}{2575 + 10.2} = 0.996 \tag{18}$$

By applying the Markov chains and the decomposition method [11], we can make an algorithm which will be implemented in C++ programming.
The availability for all the system (8 machines) is presented in the following figure:

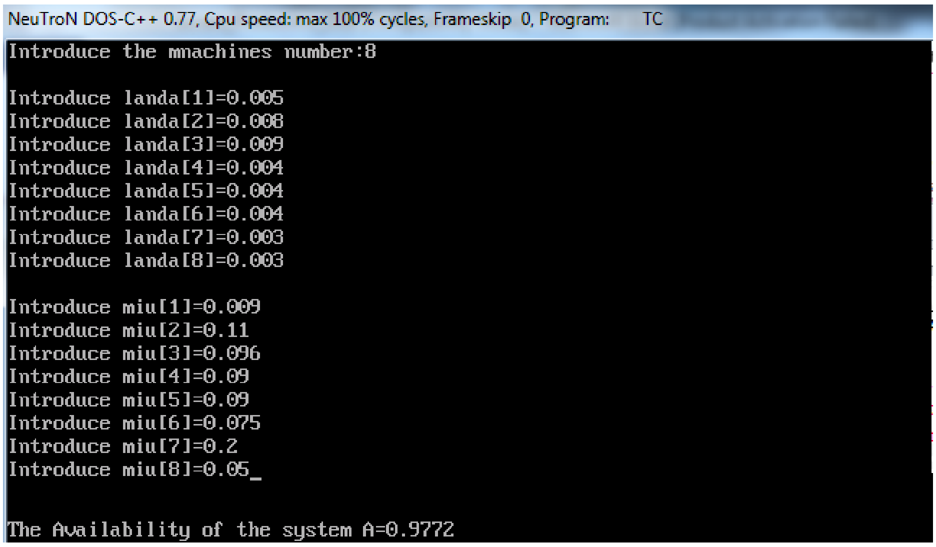


Fig. 5. Availability of the system calculated with C++

Conclusions

This paper presents a systematic approach from the set-up of Markov models to the final step of calculating the MTTF and MTTR values. The calculations steps have been presented, detailed and examined in a real case study. We developed several algorithms in order to estimate the availability, MTTF and MTTR for a workstation with two machines. These parameters can be used to improve the efficiency of the processes in a company. By applying the decomposition method and C++ programming, we found the availability of the whole system (8-machined workstation). This method can be used for any flow line to calculate its availability.

Further research can be done with a similar model with different assumptions. For example: we consider a machine M which is subject to two types of failures: failures of type F1 and of type F2. The global failure rate is λ, and with probability α the failure is of type F1, with probability $(1-\alpha)$ it is of type F2. When a failure F1 occurs, the repair is done by the operator; the repair duration is exponentially distributed with the rate μ_1. When a failure F2 occurs, the operator calls a maintenance operator. The waiting time for the maintenance operator is exponentially distributed with the rate μw. When the maintenance operator is present, the duration of the repair is exponential with the rate μ_2.

Acknowledgment

The work has been funded by the Sectorial Operational Program Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU/159/1.5/S/134398.

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