

## COLLECTIVE PHENOMENA AND CLUSTERING: FROM NUCLEAR SYSTEMS TO FUSION PLASMAS

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**Abstract.** *The present work aims at presenting new theoretical contributions to the understanding of physical systems whose strongly nonlinear intrinsic character leads to a rich variety of complex manifestations, interesting from both theoretical and practical points of view. The main theoretical difficulty is the correct identification of the degrees of freedom for each physical system under scrutiny, supplemented by the need to develop the suitable theoretical framework, or to create it where needed. It is remarkable that this enterprise has been possible both in the field of nuclear structure and in the study of the evolution of the fusion plasma turbulence. In the first part of this work, we study the interplay of collectivity and clustering within nuclear structure, where a central role is played by the two and four body correlations. In the second part, we are interested in the nonlinear processes due to the collective unstable modes in the magnetically confined plasmas. In this context, the competition between order and chaos is manifest through the generation of cuasicoherent structures on the turbulent background which have a significant influence on the transport of particles and energy.*

**Keywords:** Clustering, coherence, correlations, alpha decay, plasma turbulence

### 1. Introduction

The theoretical description of proton rich or neutron rich atomic nuclei has become an important part of modern nuclear physics, especially due to the experimental advances in the study of highly unstable nuclei.

The main investigation tool is represented by the radioactive decays, which include the phenomena induced by the strong interaction like the emission of individual particles (protons or neutrons), two-proton emission, alpha decay, heavy cluster emission and also binary and ternary fission.

There can also occur processes which are induced by the electromagnetic or weak interactions, but in what follows we will address those of nuclear origin, due to their importance in the study of exotic nuclei [1].

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The saturation property of nuclear forces implies the fact that nucleons can be "assembled" or "disassembled" relatively easily. We may consider the nuclear clustering as the study of this dynamical assembling and disassembling of nucleons which appears frequently in many problems regarding both nuclear structure and reactions. Cluster formation is a fundamental aspect of nuclear dynamics, together with the formation of the nuclear mean field.

Additionally, a unique trait (and in the same time a fundamental problem) of nuclear systems is related to the coexistence of the two types of structures, cluster and mean field, and also to the possible ways in which nuclear states may transition between shell-model type and cluster type [2].

## 2. Nuclear Correlations

The simplest nuclear correlations, those of two-body character, are directly related to the properties of the low lying collective states.

The collective excitations are described microscopically by a superposition of pair creation operators acting on a ground state which is described by a coherent state within the bosonic approximation to the Random Phase Approximation (RPA).

The coherent state in this context is defined as an exponential excitation of pair operator products acting on the ground state [3]. The ground state properties for even-even nuclei are well reproduced by the pairing interaction [4-6]. The wave function within the Bardeen-Cooper-Schrieffer (BCS) approach is also of coherent type that is an exponential excitation written in terms of creation operators acting on the ground state. The spatial distribution of the two-body density is extremely important in understanding nuclear correlations [7, 8].

In particular, in Ref. [9] the relationship between the coherence properties and chaoticity was analysed for the nuclear pairing interaction. The coherence property is characterized by the so-called Coherence Length, defined as the mean squared distance averaged over the density. For super-fluid nuclei, this averaging is performed using the pairing density. In Ref. [10, 11, 12] it was proven that this quantity has relatively large values, comparable to the nuclear size, within the nucleus; however towards the nuclear surface its behaviour is decreasing.

This picture of an extended two-nucleon cluster can be understood in terms of the Pauli blocking effect, which prevents the nucleons to cluster inside the nucleus, resulting in less bound and more spatially extended clusters. Additionally, the alpha clustering phenomenon is also related to the narrow region around the nuclear surface [1, 13], being connected to the large binding energy of an alpha particle living in a low density region [14].

As such, it is expected that the corresponding correlation length, estimated between pairs of protons and neutrons, should have significantly lower values.

We have characterized the Pairing Coherence Length for all regions in the nuclear table and we have found that it exhibits strong shell effects.

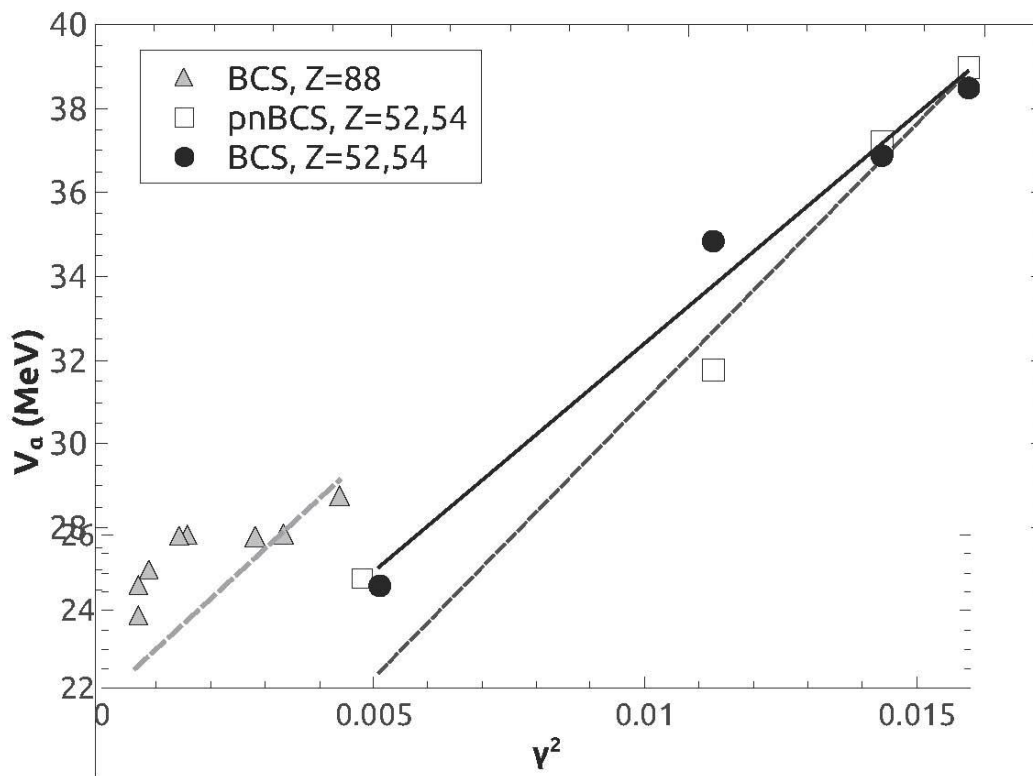
Especially, we have observed larger values for the exotic nuclei near the stability lines.

Our analysis was performed for various types of pairing interactions, in all cases the behaviour of the Coherence Length being similar.

Additionally, our results are very close to those presented in the literature for more complicated approaches like the Hartree-Fock-Bogoliubov approximation.

Also, our theoretical framework allows the description of the correlations between proton and neutron pairs, characterized by the Quartet Coherence Length.

In this case, the role played by the proton-neutron correlations is essential in modifying the properties of the Quartet Coherence Length, such that the resulting alpha Coherence Length can display a weakly oscillating behaviour around the geometrical size of an alpha particle. Its mean value of about 1.7 fm depends very weakly on the nuclear mass.



**Fig. 1.** The strength of the clustering component of the mean field versus the  $\alpha$  reduced decay width.

These aspects are intimately related to the alpha decay process, whose understanding is fundamental in explaining the properties of unstable physical systems [15], in particular for the nuclei having similar numbers of protons and neutrons where the proton-neutron correlations become significant. The first microscopic estimations within the shell-model approach for the absolute alpha decay widths have given results orders of magnitude smaller than the experimental values [17, 16].

It was understood that increasing the number of configurations leads to a significant increase of the decay width [18, 19]. However, even if a large number of shells is included in order to simulate the effects of the continuum part of the spectrum, the resulting absolute decay width is still at least two orders of magnitude too small [20, 21]. The reason for the increase of the decay width with the increase in the number of considered single particle states has turned out to be related to the clustering of high lying configurations corresponding to an alpha particle [13]. However, even if the clustering effects are described appropriately, the absolute decay width is still one order of magnitude too small [22-24].

An explanation for this behaviour may be found within a simple phenomenological model where the cluster is supposed to be formed in a potential well localized around the nuclear surface. This model predicts a linear dependence between the logarithm of the reduced decay width and the fragmentation potential.

It is remarkable that this fact is confirmed for all emission processes, including proton emission and heavy cluster emission [25]. All these considerations suggest the fact that the single particle representation must be supplemented by an attractive potential similar to that mentioned above, beyond the standard mean field of Woods-Saxon plus spin-orbit type [26]. It should be mentioned that the alpha type correlations are important for the structure of the ground state and low-lying excited states in light nuclei, [23, 27-30], as well as in heavier nuclei where a quartet quasiparticle picture may be appropriate [31, 32]. Additionally, above the doubly magic nuclei  $Z = N = 50$  there is an island of alpha decaying nuclei with  $Z \sim N$ , where the proton-neutron correlations become important and are thus able to describe nuclear structure details like the increase in collectivity in the vicinity of  $^{100}\text{Sn}$  [33]. Within our theoretical framework, in order to reproduce the alpha decay experiment collective data, we have effectively included the four body correlations at the mean field level by including an attractive Gaussian potential localized around the nuclear surface. We have found that its strength is proportional to the reduced alpha decay width, being significantly larger for the nuclei with  $Z \sim N$  than for those in the actinide region.

The remarkable fact is that these correlations have a universal character along the nuclear table and are also independent on the proton-neutron pairing interaction.

We have also evidenced a staggering effect in the formation probabilities of an alpha particle for the alpha emitting nuclei, which is a new indication of alpha clustering in medium and heavy nuclei. The nuclear landscape of manifestations is also populated by various resonance modes which are exhibited by the vast majority of nuclei, due to the collective motions of protons and neutrons. The most robust collective motion is the so-called Giant Dipole Resonance, (GDR), which may be understood microscopically as oscillations of all neutrons against all protons. This collective motion is present in all nuclei and has a position of the energy centroid which varies as  $80 \cdot A^{-1/3}$  MeV for medium-heavy nuclei.

Additionally, the GDR may be used in order to explore the properties of nuclei in extreme conditions of temperature and deformation [34]. Away from the stability line exotic collective excitations may appear [35]; the available information of these modes is still incomplete, and their true nature is still under debate.

An interesting exotic mode is the Pygmy Dipole Resonance (PDR), observed as a significant concentration of the dipole response at significantly lower energies than those associated with the GDR. At a microscopic level, this resonance may be interpreted as the result of the oscillation of the nuclear core against the excess of neutrons specific to exotic nuclei.

In light nuclei the alpha particles are influenced by this motion, leading to their oscillation against the nuclear core [36, 37].

For the nuclei above 100 Sn, the large values of the effective interaction strength which implements the four body correlations at the mean field level suggests a robust presence of alpha clusters in this region, which may have noticeable consequences on the dipole response. As such, we have investigated the collective dipole oscillations of fi, GDR and pygmy type in a schematic harmonic oscillator model by generalizing the classical approach due to Brink.

The separability of coordinates within the harmonic oscillator shell model indicates that the oscillations are of alpha-core type (in the more general case of an arbitrary dipole-dipole interaction, the normal modes are superpositions of the modes present in the decoupled case). The ratio between the Energy Weighted Sum Rule fractions exhausted by the alpha and GDR modes has been investigated for nuclei with  $N = Z$  and  $N > Z$ ; in both cases we have found a decrease of this ratio with the increase of the spectroscopic factor.

As such, we have noted upon the possibility of experimental detection of alpha type oscillations in medium nuclei above 100Sn, around 8 MeV, in competition with the GDR and PDR modes.

We have then studied a microscopic description of quasimolecular alpha-core states. We have assumed that the alpha-nucleus potential is of Gaussian type in

the nuclear region, its parameters being determined within the coupled channels approach from the continuity conditions of the logarithmic derivatives of the potential itself and of the wave functions at the boundary between the internal nuclear region and the external one.

In the latter, the wave functions may be written in terms of the partial decay widths, and the potential we assume a realistic double folding form.

We found an equilibrium radius for the internal Gaussian potential which is larger than the Mott transition point from the nucleon to alpha cluster phase in finite nuclei.

Our prediction is that the first vibrational state within the quasimolecular alpha-core potential is near the Coulomb barrier and as such the rotational band may be evidenced as a structure of peaks in the scattering cross section of alpha particles.

The dipole excitation on the state  $n = L = 1$  by beams in alpha emitting odd nuclei will bring new evidences regarding the existence of quasimolecular alpha-core structures in heavy nuclei.

### 3. Evolution of the Fusion Plasma Turbulence

Another open problem, extremely important for the physics of this century, is related to the search for an understanding of the laws that govern the systems which are found in a state of strong non-equilibrium, and the magnetically confined plasma brings many challenges in this direction [38]. Within plasma physics, a central role is played by the theory of transport phenomena; here, the starting point is the basic principles of mechanics and electrodynamics, the goal being a prediction of the macroscopic properties of the system. [39].

The main complication arises as the particle and heat transport in magnetically confined plasmas is strongly influenced by the drift turbulence generated by the gradients of temperature, density, magnetic field etc.

Within the plasma physics context, the Test Modes Approach offers a theoretical framework able to naturally include the effects of trajectory clustering on the evolution of the turbulence.

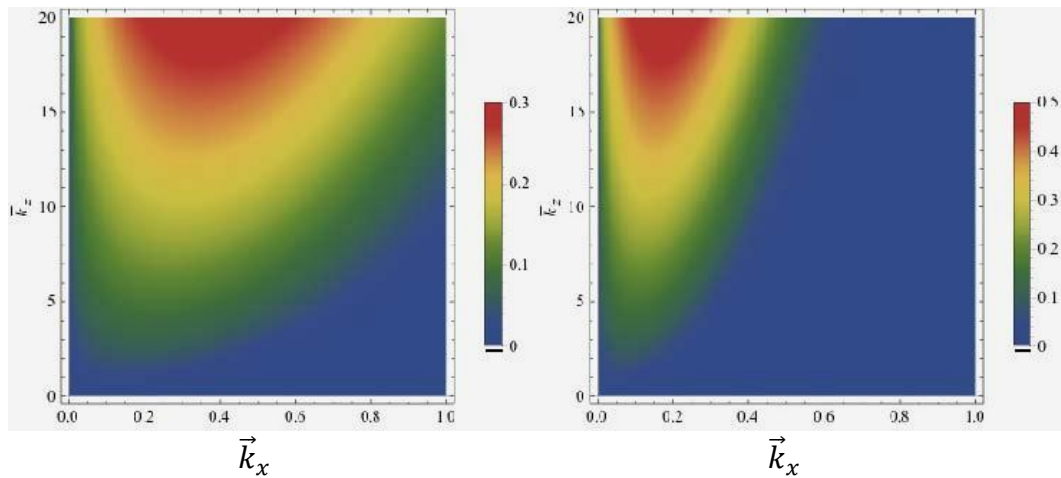
Starting from the drift kinetic equation for the guiding centres of electrons and ions in a background turbulence, we evaluate the perturbations in the distribution functions and hence in density which are generated by a given test mode.

The quasineutrality condition translated into a dispersion relation  $D(\omega, \vec{k}) = 0$ , from which the frequencies and growth rates of the various modes  $\omega = \omega_R + i\gamma = \omega(\vec{k})$  may be extracted, and as a consequence the tendencies in the evolution of the turbulence may be identified [40].

The effects of the background turbulence are included in the above mentioned description through the ion propagator, whose evaluation is done by the statistical average of ion trajectories in the background turbulence.

These may exhibit both coherent and random aspects, which appear as random large jumps and capture events.

Especially for the study these complex trajectories there have been formulations of semi-analytical statistical methods, namely the Decorrelation Trajectory Method and the Nested Subensemble Approach, in which the physical picture of the nonlinear transport processes becomes transparent.



**Fig. 2.** The growth rates of zonal flow modes for the ITG type turbulence for small values (left) and large values (right) of the diffusion coefficient.

These allow the evaluation of the average ion propagator which in turn leads to an analytical expression for the statistical distribution of trajectories corresponding to all regimes of the turbulence evolution, from the quasi-linear to the nonlinear regime.

Within the analysis of the test modes for the turbulence of the Ion Temperature Gradient type, we have reproduced the dispersion relation well known in the literature for the case of a quiescent plasma.

Surprisingly, however, we have discovered, in the quasilinear regime, not only the diffusive damping of high  $k$  modes, but also the generation of zonal flows due to the ion trajectory diffusion along the temperature gradient by the coupling between the ion propagator and the background turbulence.

This dual role of diffusion as both generating and damping of modes, relevant even in the quasilinear regime represents the main conclusion of the present analysis for the ITG turbulence.

On the other hand, in the case of the Trapped Electron Modes instability, we have evidenced in the nonlinear regime a completely different process than that present in the case of the drift turbulence. In this case, the formation of trajectory structures leads to the amplification of the destabilizing effect of the trapped electrons.

Eventually, this overcomes the diffusive damping, which in turn leads to larger growth rates, comparable to those found in the absence of turbulence.

#### 4. Conclusions

Regarding the nuclear physics aspects, the main difficulty is related to finding an appropriate description of the cluster degrees of freedom.

We have chosen to modify the mean field to accommodate the effects of four-body correlations on the alpha decay width.

A more satisfying description would involve a self-consistent approach of Hartree-Fock-Bogoliubov type, where the quasiparticles include in their structure these few-body correlations, which would lead to the natural emergence of the attractive component of the nuclear mean field in the surface region.

The results obtained within the plasma physics context allow the development of numerical modules which may be implemented within the simulation codes of transport phenomena.

Additionally, we expect to gain a better understanding of the nonlinear phenomena which determine the saturation of turbulence, the generation of zonal flows and of the plasma rotation, especially because this approach offers a different perspective with respect to the numerical simulations [41].

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