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ODD-EVEN EFFECT AND NUCLEAR INERTIA IN FISSION PROCESSES FROM THE MICROSCOPIC EQUATIONS OF MOTION

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Abstract

The time dependent pairing equations are obtained from the variational principle. The BCS occupation and vacancy amplitudes are supplied by these equations when the deformation of the nuclear system evolves in time. These equations were generalized to include the Landau-Zener promotion mechanism in superfluid systems. During the nuclear disintegrations, the single particle levels are rearranged. But, two single particle levels characterized by the same good quantum numbers cannot intersect and give rise to the so called avoided levels crossing regions. In such regions, it is possible that the nucleons promote from one level to another. By considering such mechanisms, a new dynamical pair breaking effect was evidenced. Within this formalism, the experimental fragment distribution obtained in cold fission was reproduced in the energy region were an inversion of the even-odd effect is observed. By taking into account the matrix elements of the time derivative operators in deducing the equations of motion, a new formula for the nuclear inertia was derived. This formula takes into account the dissipated energy. If the theory is particularized for adiabatic system, then the well known cranking formalism resorts.

keywords: time dependent pairing equations, pair breaking, nuclear inertia

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1 Introduction

Some results concerning the possibility of modeling the pair breaking mechanism during a large scale amplitude motion of a nuclear system, together with the modification of the effective mass due to the dissipated energy, are presented in this contribution. Both theories resort from generalized time dependent pairing equations that are deduced from the variational principle. Due to the extreme saturation of the nuclear matter, the nucleons move inside the nucleus in a mean field defined mainly by the boundaries of its surface. Therefore, it is accepted that a dynamical description can be realized by investigating the change of the nuclear shape. The nuclear shape parametrization depends on collective coordinates associated to a choice of degrees of freedom. In fission, these collective coordinates are forced to vary in order to reach a scission configuration. The way in which the nuclear matter responds to the external forces responsible for large scale amplitude deformation is described by inertia and effective mass. At the same time, by solving the Schrödinger equation for the mean field potential, it is possible to determine many intrinsic quantities, as for example eigenfunctions of nucleons and their single particle energies. During the evolution of the nuclear system, additional intrinsic excitations are produced. That is, a flow of energy and of angular momentum from the collective motion into the intrinsic one is produced, called dissipation. The Landau-Zener promotion mechanism is one way to take into account the dissipation in nuclear motion processes. The effective mass is also modified by the dissipation. In the following, the way in which a dynamical pair breaking effect is produced due to the Landau-zener promotion mechanism and the influence of the dissipation on the effective mass are discussed.

2 Formalism

The main features of the generalized time dependent equations of motion are presented in this section. Exact derivation of the formulas can be found in Refs. [1, 2, 3]. The equations of motion are obtained within the variational principle by starting with the energy functional

$$\mathcal{L} = \left\langle \varphi \left| H - i\hbar \frac{\partial}{\partial t} + H' - \vec{\Omega}\vec{J} - \lambda \hat{N} \right| \varphi \right\rangle , \tag{1.1}$$

where we denoted with $\vec{J}$ (in $\hbar$ units) the total angular momentum. $\vec{\Omega}$ is the angular velocity. As usual, λ is a notation for the Fermi energy that results from the BCS equations, and $\hat{N} = \sum_{k>0}(a_k^+ a_k + a_{\bar{k}}^+ a_{\bar{k}})$ is the particle number operator.

A trial many-body function $|\varphi\rangle$ expanded as a superposition of time dependent BCS seniority-zero and seniority-two wave functions is considered

$$|\varphi(t)\rangle = c_0(t)|\phi_{\mathrm{BCS}}\rangle + \sum_j c_{jj}(t)\alpha_j^+\alpha_{\bar{j}}^+|\phi_{\mathrm{BCS}}\rangle + \sum_{j,l} c_{jl}(t)a_j^+a_{\bar{l}}^+|\phi_{\mathrm{BCS}(jl)}\rangle, \tag{1.2}$$

where the Bogoliubov wave function is

$$|\phi_{\rm BCS}\rangle = \prod_k (u_k(t) + v_k(t) a_k^+ a_{\bar{k}}^+)|0\rangle. \tag{1.3}$$

The Hamiltonian with pairing residual interactions is given by

$$H(t) = \sum_{k>0} \epsilon_k(q)(a_k^+ a_k + a_{\bar{k}}^+ a_{\bar{k}}) - G(q) \sum_{k,i>0} a_k^+ a_{\bar{k}}^+ a_i a_{\bar{i}}. \tag{1.4}$$

The interaction that allows to simulate a promotion mechanism contribution in superfluid systems similar to the Landau-Zener effect is defined as

$$\begin{array}{c} H' = \sum_{\Omega,m,m'} h_{\Omega,m,m'} \alpha^+_{\Omega,m(\Omega,m')} \alpha_{\Omega,m'(\Omega,m)} \\ \times \prod_{\Omega',m''} \alpha_{\Omega',m''(\Omega,m')} a^+_{\Omega',m''} a_{\Omega',m''} \alpha^+_{\Omega',m''(\Omega,m)}. \end{array} \tag{1.5}$$

Due to the interaction $h_{\Omega,m,m'}$ that is produced between two single particle orbitals that have the same good quantum number in the avoided crossing regions, a nucleon can be promoted from one single particle level to another.

After performing the variation of the energy functional, by neglecting the angular momenta and the matrix elements of the time derivative, the following system of differential equations emerges:

$$i\hbar\dot{\rho}_{k(0)} = \kappa_{k(0)}\Delta^*_{k(0)} - \kappa^*_{k(0)}\Delta_{k(0)}, \tag{1.6}$$

$$i\hbar\dot{\rho}_{k(jl)} = \kappa_{k(jl)}\Delta^*_{k(jl)} - \kappa^*_{k(jl)}\Delta_{k(jl)}, \tag{1.7}$$

$$\begin{array}{c} i\hbar\dot{\kappa}_{k(0)} = \left(2\rho_{k(0)} - 1\right)\Delta_{k(0)} + 2\kappa_{k(0)}\left(\epsilon_k - sN_{i_k}\lambda\right) \\ -2G_{kk}\rho_{k(0)}\kappa_{k(0)}, \end{array} \tag{1.8}$$

$$\begin{array}{c} i\hbar\dot{\kappa}_{k(jl)} = \left(2\rho_{k(jl)} - 1\right)\Delta_{k(jl)} + 2\kappa_{k(jl)}\left(\epsilon_k - sN_{i_k}\lambda\right) \\ -2G_{kk}\rho_{k(jl)}\kappa_{k(jl)}, \end{array} \tag{1.9}$$

$$i\hbar\dot{P}_0 = \sum_{l,j\neq l} h_{lj}(S^*_{0jl} - S_{0jl}) \tag{1.10}$$

$$i\hbar\dot{P}_{jl} = h_{lj}(S_{0jl} - S^*_{0jl}) \tag{1.11}$$

$$\begin{array}{c} i\hbar\dot{S}_{0jl} = S_{0jl}(\bar{E}_0 - \bar{E}_{jl}) + S_{0jl}\left(\sum_{k\neq j,l} T_{k(jl)} - \sum_k T_{k(0)}\right) \\ + \sum_{\{mn\}\neq\{jl\}} h_{mn}S_{mnjl} + h_{jl}(P_{jl} - P_0) \end{array} \tag{1.12}$$

$$\begin{array}{c} i\hbar\dot{S}_{mnjl} = S_{mnjl}(\bar{E}_{mn} - \bar{E}_{jl}) + S_{mnjl}\left(\sum_{k\neq m,n} T_{k(mn)} - \sum_{k\neq j,l} T_{k(jl)}\right) \\ + h_{mn}S_{0jl} - h_{jl}S^*_{0mn} \end{array} \tag{1.13}$$

where j, k, l, m, n label the single particle levels in the active pairing space. The sign $s = \pm 1$ is introduced in oder to have a dynamic projection of the number of particles in the final fragments. We used the following notations:

$$\begin{array}{c} \Delta_{k(0)} = \sum_{k'} \kappa_{k'(0)} G_{kk'};\ \Delta_{k(jl)} = \sum_{k' \neq j,l} \kappa_{k'(jl)} G_{kk'}; \\ \kappa_{k(0)} = u_{k(0)} v_{k(0)};\ \rho_{k(0)} = \mid v_{k(0)} \mid^2; \\ \kappa_{k(jl)} = u_{k(jl)} v_{k(jl)};\ \rho_{k(jl)} = \mid v_{k(jl)} \mid^2; \\ P_0 = \mid c_0 \mid^2;\ P_{jl} = \mid c_{jl} \mid^2; \\ S_{0jl} = c_0 c_{jl}^*;\ S_{mnjl} = c_{mn} c_{jl}^*. \end{array} \tag{1.14}$$

As usual, Δ_γ denote the gap parameter. The solutions of the system are the single particle densities ρ_γ, the pairing moment components κ_γ, the probabilities to have a given seniority configuration P_γ, and the moment components between configurations $S_{\gamma\gamma'}$. The relations (1.6)-(1.9) are the well known time dependent paring equations previously deduced in Refs. [4, 5].

By introducing the matrix elements of the time derivative operator, and considering that the process is slow enough that the seniority zero configuration is dominant, new formulas for the effective mass and for the moments of inertia can be deduced. The elements of the mass tensor are:

$$B_{\nu\mu} = B_{\nu\mu}^N + B_{\nu\mu}^D \tag{1.15}$$

where

$$\begin{array}{c} B_{\nu\mu}^N = 2\hbar^2 \sum_{m,n\neq m} \left\{ (E_{mn} - E_0) \left| \frac{\kappa_m \sqrt{\rho_m} |\kappa_n|}{|\kappa_m| \sqrt{\rho_n}} - \frac{\kappa_n \sqrt{\rho_n} |\kappa_m|}{|\kappa_n| \sqrt{\rho_m}} \right|^2 \right. \\ \times \left\langle m \left| \frac{\partial H}{\partial q_\nu} \right| n \right\rangle \left\langle n \left| \frac{\partial H}{\partial q_\mu} \right| m \right\rangle |P_{mn00}|^2 \\ \left. / \left[\left(E_{mn} - \sum_{k\neq m,n} T_{k(mn)} - E_0 + \sum_k T_k \right)^2 (\epsilon_m - \epsilon_n)^2 \right] \right\} \end{array} \tag{1.16}$$

$$\begin{array}{c} B_{\nu\mu}^D = \hbar^2 \sum_m \left[\left(\frac{\kappa_m}{\rho_m} \frac{\partial \rho_m}{\partial q_\nu} - \frac{\kappa_m}{\kappa_m^*} \frac{\partial \kappa_m^*}{\partial q_\nu} \right) \left(\frac{\kappa_m^*}{\rho_m} \frac{\partial \rho_m}{\partial q_\mu} - \frac{\kappa_m^*}{\kappa_m} \frac{\partial \kappa_m}{\partial q_\mu} \right) \right. \\ \left. + \left(\frac{\kappa_m}{\rho_m} \frac{\partial \rho_m}{\partial q_\mu} - \frac{\kappa_m}{\kappa_m^*} \frac{\partial \kappa_m^*}{\partial q_\mu} \right) \left(\frac{\kappa_m^*}{\rho_m} \frac{\partial \rho_m}{\partial q_\nu} - \frac{\kappa_m^*}{\kappa_m} \frac{\partial \kappa_m}{\partial q_\nu} \right) \right] / \left(E_{mm} - E_0 \right). \end{array} \tag{1.17}$$

The collective moments of inertia are:

$$I_i = 2 \sum_{m,n\neq m} \frac{(E_{mn} - E_0) \left| \frac{\kappa_m \sqrt{\rho_m} |\kappa_n|}{|\kappa_m| \sqrt{\rho_n}} - \frac{\kappa_n \sqrt{\rho_n} |\kappa_m|}{|\kappa_n| \sqrt{\rho_m}} \right|^2 \langle m | J_i | n \rangle^2 |P_{mn00}|^2}{\left(E_{mn} - \sum_{k\neq m,n} T_{k(mn)} - E_0 + \sum_k T_k \right)^2}. \tag{1.18}$$

In the previous equations, the mass parameters depend on the solutions of the time dependent pairing equations ρ_k and κ_k. The dissipated energy of the system can be obtained by evaluating the total energy of the system with these solutions. Therefore, these parameters depend on the dissipated energy accumulated in the system. Moreover, By particularizing the formulas through the BCS solutions for adiabatic systems, the well known formulas given by the cranking model are retrieved.

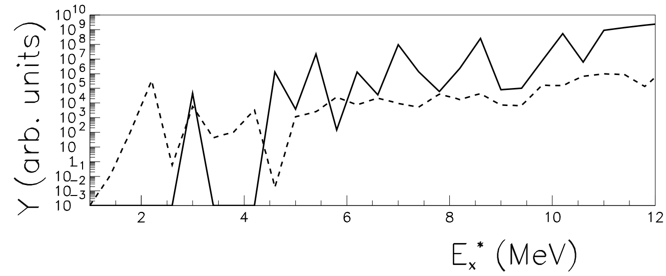

Figure 1: Dependence of the fission yields for the mass fragmentation 90/144 as function of the excitation energy E^*. The full line is the yield of the even-even fragments ^{90}Kr+^{144}Ba and the dashed line is for the odd-odd fragments ^{90}Rb+^{144}Cs.

3 Results

A very strange phenomenon was observed in cold fission. For excitation energies lower than 4 MeV, an inversion of the even-odd effect in the mass distribution was evidenced experimentally [6, 7, 8]. At these energies, the odd-odd mass partitions dominates the fission of an even-even parent nucleus. This phenomenon can be explained within the microscopic equations of motion (1.6)-(1.13). We are interested to determine the yields as function of the excitation energy in the cold fision of ^{234}U for two fragmentation configuration, namely ^{90}Rb+^{144}Cs (odd-odd) and ^{90}Kr+^{144}Ba (even-even) that belong to the same mass partition 144/90. Experimental data are available for the energy dependence of these yields. First of all, a fission trajectory beginning from the ground state of the parent nucleus and reaching the scission configuration was evaluated by using the least action principle. The generalized pairing time dependent equations were solved along this deduced fission trajectory for different values of the tunneling velocity. The tunneling or internuclear velocity is defined as the time derivative of the collective distance between the centers of the emerging fragments. We observed that, for low collective velocities, the dissipated energy is very small, the fissioning system being almost adiabatic at scission. In this case, the dissipated energy is less than 1 MeV. When the tunneling velocity increases towards very large values, the dissipation energy is close to 12-14 MeV and reaches a plateau. In the same time, at low velocities we obtain only odd-odd fission fragments, P_0 being almost 0. At intermediate collective velocities that lead to dissipated energies of the order of 4 MeV, the even-even fragment yields begin to dominate the odd-odd ones. The energy dependence of the yields is displayed in Fig. 1. So, by solving the equations of motions we succeeded to describe precisely the experimental trends.

Performing the variation for the same energy functional as that determining the time dependent paring equations, new formulas for the inertia are deduced for the effective masses (1.15) and for the moments of inertia (1.18). These formulas include the dissipated energy produced during the evolution of the nuclear system, from the parent ground state up to scission. We observed that for low internuclear velocities, when the nuclear system behaves as adiabatic, the values given by the new formulas reproduce exactly those given by the adiabatic cranking model [9]. For intermediate values of the internuclear velocities, the effective masses and the moments of inertia decrease with about 10 %. For even larger values of the internuclear velocities, close to the limit of validity of the model, the inertia increases asymptotically towards very large values. So, the collective velocity should decrease. This asymptotic behaviour prevents the appearance of negative values of the inertia for large values of the collective velocities. We consider that large values of the internuclear velocity are about 1/10 of the Fermi velocity of the nucleon. The non adiabatic inertia has a similar shell structure as the adiabic one.

4 Conclusions

We proposed a microscopic model for the explanation of the inversion of the odd-even effect in the mass distribution of cold fission. This model takes into account the Landau-Zener effect. In a superfluid system, the Landau-Zener promotion mechanism is responsible for a new dynamical pair breaking effect. This effect is incorporated in a new set of time dependent pairing equations. The experimental yield distributions were reproduced. Concerning the investigation of the mass parameters, we concluded that inertia strongly depends on the dissipated energy accumulated in the fissioning system during the large scale amplitude motion. It can be postulated that the system should adjust its collective velocity in order to prevent negative values of the effective mass.

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