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# INVESTIGATING THE STRUCTURE OF HEAVY AND SUPERHEAVY NUCLEI THROUGH α-EMISSION

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## 1. Introduction

This thesis presents original contributions to the study of nuclear structure within the region of heavy and super heavy nuclei by means of the α-decay process. Chapter 2, based on Ref. [1], presents the basic theoretical ideas behind the α-nucleus interaction. Chapter 3 is dedicated to the study of nuclear structure by means of the Coherent State Model (CSM) for even-even and odd-mass nuclei (Refs. [2, 3]). Within are analyzed the ground state (g.s.) bands of quadrupole deformation for even-even nuclei and the rotational bands of odd mass nuclei built upon a single particle (s.p.) orbital of angular momentum projection $\Omega \neq \frac{1}{2}$, the latter for the case of favored α-transitions. Both energy levels and the values of the reduced rates of quadrupole electric transitions ($B$ ($E2$) values) are calculated. Chapter 4 (Refs. [2, 3]) treats the α-spectroscopy for transitions to the states previously described. The total α-decay half-life and the intensities of α-transitions to excited states are reproduced for all available data by means of an interaction having a monopole and quadrupole (QQ) component, together with the coupled channels method. Predictions are made for the completion of partial data sets as well as nuclei where experimental investigations are still quite limited, as for example superheavy nuclei.

## 2. The Coupled Channels Formalism

The general α-decay process under consideration is

$$P(I_P) \rightarrow D(I) + \alpha(l), \tag{2.1}$$

where $I_P$ denotes the spin and parity of the parent nucleus, I represents the spin and parity of the daughter nucleus and l is the orbital angular momentum of the α-particle. We consider a cluster wave function [4] with the total spin of the initial state.

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$$\Psi_{I_P M_P}(\xi_D, \mathbf{R}) = \sum_{c=(I,l)} \frac{f_c(R)}{R} \mathcal{Y}^{(c)}_{I_P M_P}(\xi_D, \hat{R}) , \quad (2.2)$$

where $\mathcal{Y}^{(c)}_{I_P M_P}(\xi_D, \hat{R})$ are the so called core-angular harmonics that form a complete orthonormal set. The dynamics of the α-daughter nucleus system is described by the stationary Schrödinger equation

$$H\Psi_{I_P M_P}(\xi_D, \mathbf{R}) = Q_\alpha \Psi_{I_P M_P}(\xi_D, \mathbf{R}), \quad (2.3)$$

where $Q_\alpha$ is the energy of the decay process. The Hamiltonian

$$H = -\frac{\hbar^2}{2\mu}\nabla^2_R + H_D(\xi_D) + V(\xi_D, \mathbf{R}) , \quad (2.4)$$

includes a kinetic term, dependent on the reduced mass μ, a term that described the dynamics of the daughter nucleus $H_D(\xi_D)$ and the α-daughter interaction which decomposes in a term having spherical symmetry and a deformed component. Using the orthonormality condition for angular harmonics in the superposition (2.2), one obtains through standard methods a system of coupled differential equations for the radial components [4]. It is worth noting that, at large distances where only the Coulomb field is present the system of equations becomes decoupled, its solutions having the asymptotic expression

$$f_c(\chi_c, \rho_c) \to N_c H_l^{(+)}(\chi_c, \rho_c) \quad (2.5)$$

as functions of the spherical Coulomb-Hankel wave dependent on the Coulomb parameter and the reduced radius. Using the continuity equation, one finds that the total decay width is a sum of partial widths [4]

$$\begin{aligned} \Gamma &= \sum_c \Gamma_c = \sum_c \hbar v_c \lim_{R\to\infty} |f_c(R)|^2 \\ &= \sum_c \hbar v_c |N_c|^2 , \end{aligned} \quad (2.6)$$

as function of the center of mass velocity at infinity for each channel *c*.

## 3 The Structure of α-Emitters

The CSM model was developed in Refs. [5, 6] as a tool used in describing the spectra of vibrational, transitional and rotational nuclei in a unified framework. This model treats the vibrations of a deformed nuclear surface with the help of a coherent superposition of phonons. An introduction to these ideas can be consulted in Refs. [7, 8]. Following that, the model was considerably extended in Refs. [9, 10] in order to allow the description of low and high spin nuclear states, including isospin degrees of freedom (a review of these results can be consulted in Ref. [11]).

The intrinsic wave function of an even-even nucleus with axial deformation is given by a coherent superposition of quadrupole bosonic operators $b_{2\mu}$ with $\mu = 0$, acting upon the vacuum state

$$|\psi_g\rangle = e^{d\left(b_{20}^{\dagger} - b_{20}\right)}|0\rangle \tag{2.1}$$

specified by a deformation parameter d proportional to the static quadrupole deformation [9].

The physical states defining the ground band are obtained by angular momentum projection

$$|\varphi_J^{(g)}\rangle = \mathcal{N}_J^{(g)} \hat{P}_{M0}^J |\psi_g\rangle \tag{2.2}$$

The simplest estimation of the energy spectrum of the ground band is calculated as

$$\begin{aligned} E_J(d) &= A_1 \left[\langle\varphi_J^{(g)}|\hat{N}|\varphi_J^{(g)}\rangle - \langle\varphi_0^{(g)}|\hat{N}|\varphi_0^{(g)}\rangle\right] \\ &= A_1 d^2 \left[\mathcal{I}_J(d) - \mathcal{I}_0(d)\right] \;, \end{aligned} \tag{2.3}$$

where $\hat{N}$ is the boson number operator, and the indicated functions are completely determined by the deformation parameter.

Let us notice that, for small values of d, the energy spectrum has a vibrational character $E_J \sim A_1 J$, while for large values it has a rotational shape $E_J \sim A_1 J(J + 1)$ [5]. A single parameter description of the CSM Hamiltonian leads to a universal dependence on the deformation parameter for the energy level ratios.

In the case of a nucleon coupled to a coherent state core, the energies of a rotational band built upon a single particle state of fixed angular momentum projection $\Omega \neq \frac{1}{2}$ can be estimated from the matrix element [12]

$$\begin{aligned} \langle IM|H|IM\rangle &= A_1 d^2 f_{j\Omega I} - d\left(N + \frac{3}{2}\right) \times \\ &\times \; \langle j2;\Omega 0|j\Omega\rangle\langle j2;\frac{1}{2}0|j\frac{1}{2}\rangle . \end{aligned} \tag{2.4}$$

## 4 The Fine Structure of the α-Emission Spectrum

The ultimate goal of this investigation consists in the calculation of the decay intensities

$$\Upsilon_J = \log_{10}\frac{\Gamma_0}{\Gamma_J}, \tag{2.1}$$

which can also be labeled as $\gamma_i$, i = 1, 2, 3 to designate transitions to the first, second and third states of a collective structure.

This is done by tweaking the strength of the deformed component found in the interaction term of the Hamiltonian (2.4), such that the solutions entering Eq. (2.6) match experimental decay widths.

This strength is corellated with the nuclear deformation ([2, 3]) as a straight line of equation

$$C = C_0 \left(1 - \sqrt{\frac{2}{7}} a_\alpha d\right), \tag{2.2}$$

and thus its value can be inferred from nuclear spectroscopy data.

The parameters $C_0$ and $a_\alpha$ _ are generally determined by reproducing the lowest experimental intensities. The accuracy of this method is best tested by comparing experimental and theoretical results by means of the hindrance factors

$$\log_{10} \mathrm{HF}_i = \Upsilon_i^{\mathrm{exp}} - \Upsilon_i^{\mathrm{pred}}. \tag{2.3}$$

In the case of even-even emitters in the actinides region, one obtains the results shown in Fig. 4.1.

A linear correlation between the interaction strength and deformation parameter allows for a simultaneous description of all available intensities with reasonable accuracy.

For favoured $\alpha$-transitions in odd-mass actinides, where the odd nucleon does not change state after the transition, the results are shown in Fig. 4.2. In this situation it is usually possible to reproduce $\Upsilon_1$ exactly, which furnishes good estimates of the higher intensities.

In conclusion, by properly inserting the nuclear structure data in the scattering problem via the deformation parameter, it is possible to describe the fine structure of the $\alpha$-emission spectrum.

Thus, one achieves a unified description of low-energy levels, electromagnetic transitions and fine structure emission data for even-even and odd-mass nuclei, the latter in the case of favored transitions. A number of predictions can be performed on the basis of these results, concerning either the completion of partial energy or decay data sets for actinides or estimations concerning the structure and emission properties of superheavy nuclei.

Further developments of the theory are required in order to describe collective structures in odd-mass nuclei built upon a single particle angular momentum projection $\Omega = \frac{1}{2}$, and the case of unfavored $\alpha$-transitions, where the odd particle changes state following the decay.

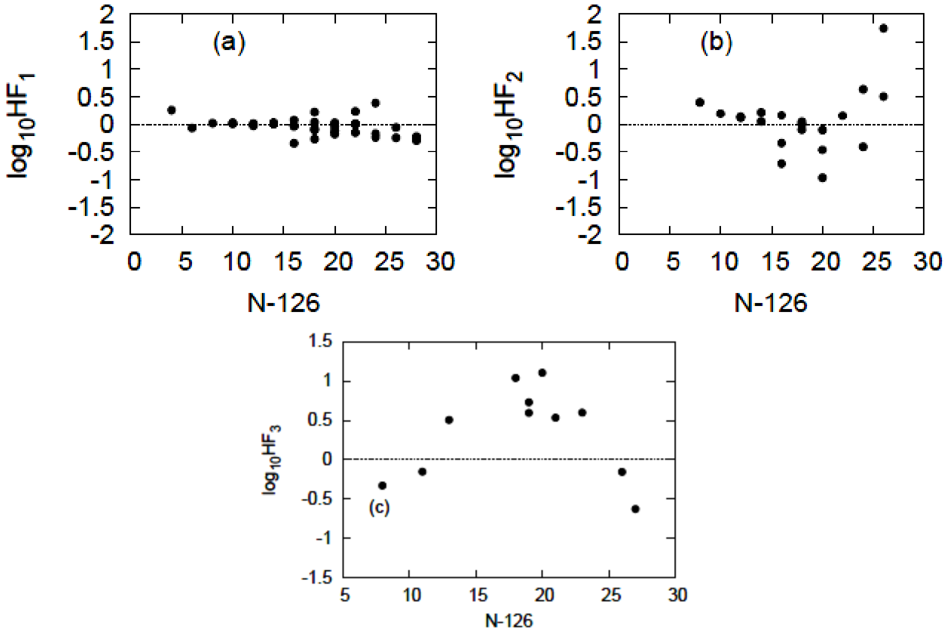


**Fig. 4.1.** Logarithm of the hindrance factors as function of the neutron number above the closed shell at $N = 126$, in the case of even-even actinides.

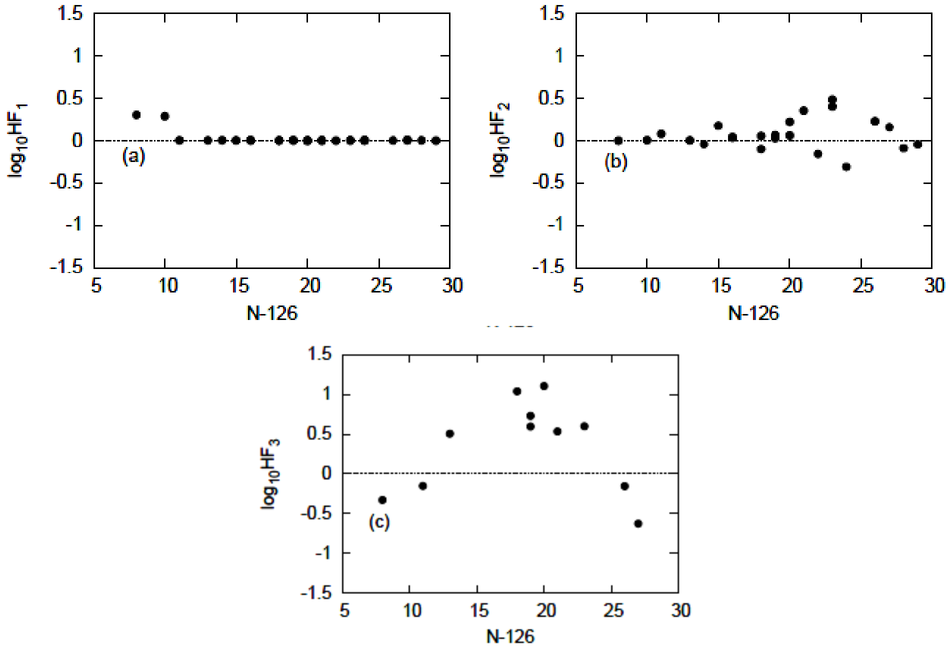


**Fig. 4.2:** Logarithm of the hindrance factors as function of the neutron number above the closed shell at $N = 126$, in the case of favored transitions in odd-mass actinides.

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