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Eigenvalues of $-(\Delta_p + \Delta_q)$ under a Robin-like boundary condition*

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Dedicated to Professor Viorel Barbu on his 75^{th} anniversary

Abstract

Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$, be a bounded open set with smooth boundary. Consider in Ω the equation $-\Delta_p u - \Delta_q u = \lambda |u|^{p-2}u$ subject to a Robin-like boundary condition involving a positive constant α, where $p, q \in (1, \infty)$, $p \neq q$, and $\lambda \in \mathbb{R}$. We show that there is no eigenvalue λ of the above problem in the interval $(-\infty, \lambda_R]$, where $\lambda_R := \inf \{ \int_\Omega |\nabla v|^p \, dx + \alpha \int_{\partial\Omega} |v|^p \, ds;\ v \in W^{1,\max\{p,q\}}(\Omega),\ \int_\Omega |v|^p \, dx = 1 \}$, while any $\lambda \in (\lambda_R, \lambda^*)$ is an eigenvalue of this problem, where $\lambda^* := \alpha m_{N-1}(\partial\Omega)/m_N(\Omega)$. Note that the case $p \neq q$ investigated here is complementary to the homogeneous case $p = q$ for which the set of eigenvalues is completely known only if $p = q = 2$.

MSC: 35J60, 35J92, 49R05

keywords: Robin-like eigenvalue problem, p-Laplacian, Sobolev space, Nehari type manifold, variational methods.

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