Ann. Acad. Rom. Sci.
Ser. Math. Appl.
ISSN 2066-6594 Vol. 8, No. 2/2016

Toeplitz operators on $L^2_a(\mathbb{C}_+)^*$

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Abstract

In this paper we show that if $a \in \mathbb{D}$ and R and S are two bounded linear operators from $L^2_a(\mathbb{C}_+)$ into itself, such that $R\mathcal{T}_{\psi_1}S = \mathcal{T}_{\psi_1 \circ t_a}$ for all $\psi_1 \in L^\infty(\mathbb{C}_+)$, then there exist $\alpha, \beta \in \mathbb{C}$ such that $R = \alpha V_a, S = \beta V_a$ and $\alpha\beta = 1$. Here $t_a(s) = \frac{-ids+(1-c)}{(1+c)s+id}$ and $V_a f = (f \circ t_a)l_a, f \in L^2_a(\mathbb{C}_+)$ where $l_a(s) = \frac{1-|a|^2}{((1+c)s+id)^2}$. Let $\mathcal{A} = \left\{\mathcal{T}_\phi : \phi \in C^\infty_c(\mathbb{C}_+)\right\}$ and $\mathfrak{T} = Cl\ \mathcal{A}$, where $Cl\ \mathcal{A}$ denotes the closure of $\mathcal{A}$ in $\mathcal{L}(L^2_a(\mathbb{C}_+))$. Let $\mathfrak{A}$ be the smallest closed algebra generated by the Toeplitz operators $\mathcal{T}_\phi, \phi \in L^\infty(\mathbb{C}_+)$ in $\mathcal{L}(L^2_a(\mathbb{C}_+))$. In this paper we further show that the set of all Toeplitz operators defined on the Bergman space $L^2_a(\mathbb{C}_+)$ of the right half plane with essentially bounded symbols is dense in the strong operator topology in the space of all bounded linear operators. As applications of these results we establish that there is no bounded projection from $\mathcal{L}(L^2_a(\mathbb{C}_+))$ onto $\mathfrak{T}$ and if $\Phi_0 : \mathfrak{A} \to \mathcal{L}(L^2_a(\mathbb{C}_+))$ is a linear isometry such that, for each pair of vectors $f, g \in L^2_a(\mathbb{C}_+)$,

$$\sup\{|\langle \Phi_0(A)f, g\rangle| : A \in \mathfrak{A} \text{ and } ||A|| = 1\} = ||f||||g||,$$

then their exits a unique extension of Φ_0 to a linear isometry, Φ, mapping $\mathcal{L}(L^2_a(\mathbb{C}_+))$ into itself.

MSC: 30H20, 47B35

keywords: Right half plane; Bergman space; Toeplitz operators; strong operator topology; bounded projection.

*Accepted for publication on May 5-th 2016

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