The Integral Version of Popoviciu's Inequality on the Real Line*

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Dedicated to the memory of Tiberiu Popoviciu, on the occasion of his 110th birthday anniversary

Abstract

T. Popoviciu has proved in 1965 an interesting characterization of the convex functions of one real variable, relating the arithmetic mean of its values and the values taken at the barycenters of certain subfamilies of the given family of points. The aim of our paper is to prove an integral analogue in the framework of absolutely continuous probability measures on the real line.

MSC: Primary 26A51; Secondary 26D15

keywords: Popoviciu's inequality, convex function, barycenter

1   Introduction

Fifty one years ago Tiberiu Popoviciu [11] published a striking result concerning the averaging properties of convex functions. Its essence is as follows: