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The space $l_\infty(X)$*

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Abstract

In this paper we have shown that the space $c_0^{n\times n}$ cannot be complemented in $l_\infty^{n\times n}$ and $c_0(H)$ cannot be complemented in $l_\infty(H)$ where H is a Hilbert space. Further, extending these results we show that if X is a Banach space then $c_0(X)$ cannot be complemented in $l_\infty(X)$.

MSC: 46B25, 46B45,46C07

keywords: Hilbert space, Banach space, projection operator, contraction, Schauder basis.

1 Introduction

A closed subspace M of a Banach space X is said to be complementd in X if and only if there exists a bounded linear projection from X onto M. It is not difficult to see that if M is complemented by the closed subspace N, then there exists a $c > 0$ such that $\|m + n\| \geq c\|m\|$ for all $m \in M$ and $n \in N$. Murray [12] showed that $l_p, p > 1, p \neq 2$ has subspaces that cannot be complemented. Philips [14] and Lindenstrauss and Tzafriri [11] proved that c_0 cannot be complemented in l^∞. In fact, Lindenstrauss and Tzafriri [11] established that every infinite dimensional Banach space which is not isomorphic to a Hilbert space contains a closed subspace that cannot be complemented. Similar results were also proved in [2],[4] [6],[15],[17] and [20]. Pelczynski [13] showed that complemented subspaces of l_1 are isomorphic to

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