

Higher Order Boundary Value Problem for Impulsive Differential Inclusions*

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Abstract

In this paper, we present some existence results for the higher order impulsive differential inclusion:

$$\left\{ \begin{array}{l} x^{(n)}(t) \in F(t, x(t), x'(t), \dots, x^{(n-1)}(t)), \text{ a.e. } t \in J = [0, \infty), t \neq t_k, \\ \hspace{15em} k = 1, \dots, \\ \Delta x^{(i)}|_{t=t_k} = I_{ik}(x(t_k), x'(t_k), \dots, x^{(n-1)}(t_k)), \text{ } i = 0, 1, \dots, n-1, \\ \hspace{15em} k = 1, \dots, \\ x^{(i)}(0) = x_{0i}, \text{ } (i = 0, 1, \dots, n-2), \text{ } x^{(n-1)}(\infty) = \beta x^{(n-1)}(0), \end{array} \right.$$

where $F : \mathbb{R}_+ \times E \times E \times \dots \times E \rightarrow \mathcal{P}(E)$ is a multifunction, $x_{0i} \in E, i = 0, 1, \dots, n-1, 0 = t_0 < t_1 < \dots < t_m < \dots, \lim_{k \rightarrow \infty} t_k = \infty, I_{ki} \in C(E \times \dots \times E, E) (i = 1, \dots, n-1, k = 1, \dots), \Delta x|_{t=t_k} = x(t_k^+) - x(t_k^-), x(t_k^+) = \lim_{h \rightarrow 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h \rightarrow 0^+} x(t_k - h)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively, $x^{(n-1)}(\infty) = \lim_{t \rightarrow \infty} x^{(n-1)}(t)$, and $(E, |\cdot|)$ is real separable Banach space.

We present some existence results when the right-hand side multi-valued nonlinearity can be either convex or nonconvex.

MSC: 34K45, 34A60, 47D62, 35R12

* Accepted for publication in revised form on August 10-th, 2015

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