

Methods and Algorithms for Approximating the Gamma Function and Related Functions. A survey. Part II: Completely monotonic functions*

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Abstract

In this survey we present our recent results on analysis of gamma function and related functions. The results obtained are in the theory of asymptotic analysis, approximation of gamma and polygamma functions, or in the theory of completely monotonic functions. In the second part of this survey we show how the theory of completely monotonic functions can be used to establish sharp bounds for gamma and related functions.

MSC: 33B15; 26D15; 11Y60; 41A60; 41A25; 34E05

Keywords: gamma function; digamma function; polygamma functions; approximations; asymptotic series; inequalities; monotonicity; complete monotonicity; Stirling formula; Burnside formula; Schuster formula; Wallis' ratio; Kazarinoff's inequality; Minc-Sathre ratio.

1 Introduction and Motivation

By a completely monotonic function on an interval I we mean a function $z : I \rightarrow \mathbb{R}$ which admits derivatives of any order and satisfies the following

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