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Methods and Algorithms for Approximating the Gamma Function and Related Functions. A survey. Part I: Asymptotic Series*

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Abstract

In this survey we present our recent results on analysis of gamma function and related functions. The results obtained are in the theory of asymptotic analysis, approximation of gamma and polygamma functions, or in the theory of completely monotonic functions. The motivation of this first part is the work of C. Mortici [Product Approximations via Asymptotic Integration Amer. Math. Monthly 117 (2010) 434-441] where a simple strategy for constructing asymptotic series is presented. The classical asymptotic series associated to Stirling, Wallis, Glaisher-Kinkelin are rediscovered. In the second section we discuss some new inequalities related to Landau constants and we establish some asymptotic formulas.

MSC: 26D15; 11Y60; 41A60; 41A25; 34E05

Keywords: gamma function; digamma function; Bernoulli numbers; approximations; asymptotic series; monotonicity; Glaisher-Kinkelin constant; Landau constants; Euler-Mascheroni constant; convergence; speed of convergence

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