

# SOME OPEN PROBLEMS CONCERNING THE CONVERGENCE OF POSITIVE SERIES\*

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## Abstract

We discuss some old results due to Abel and Olivier concerning the convergence of positive series and prove a set of necessary conditions involving convergence in density.

MSC: 37A45, 40A30, 40E05

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## 1 Introduction

Understanding the nature of a series is usually a difficult task. The following two striking examples can be found in Hardy's book (17), *Orders of infinity*: the series

$$\sum_{n \geq 3} \frac{1}{n \ln n (\ln \ln n)^2}$$

converges to 38.43..., but does it so slow that one needs to sum up its first  $10^{3.14 \times 10^{86}}$  terms to get the first two exact decimals of the sum. In the same time, the series

$$\sum_{n \geq 3} \frac{1}{n \ln n (\ln \ln n)}$$

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