SOME OPEN PROBLEMS CONCERNING THE CONVERGENCE OF POSITIVE SERIES*

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Abstract

We discuss some old results due to Abel and Olivier concerning the convergence of positive series and prove a set of necessary conditions involving convergence in density.

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1 Introduction

Understanding the nature of a series is usually a difficult task. The following two striking examples can be found in Hardy's book (17), *Orders of infinity*: the series

$$\sum_{n \geq 3} \frac{1}{n \ln n \, (\ln \ln n)^2}$$

converges to 38.43..., but does it so slow that one needs to sum up its first $10^{3.14 \times 10^{86}}$ terms to get the first two exact decimals of the sum. In the same time, the series

$$\sum_{n \geq 3} \frac{1}{n \ln n \, (\ln \ln n)}$$