Ann. Acad. Rom. Sci.
Ser. Math. Appl.
ISSN 2066-5997 Vol. 6, No. 1/2014

OPTIMAL CONTROL OF AN OBLIQUE DERIVATIVE PROBLEM*

Daniel Wachsmuth† Gerd Wachsmuth‡

Abstract

We investigate optimal control of an elliptic partial differential equation (PDE) with oblique boundary conditions. These boundary conditions do not lead directly to a weak formulation of the PDE. Thus, the equation is reformulated as a variational problem. Existence of optimal controls and regularity of solutions is proven. First-order optimality conditions are investigated. The adjoint state is interpreted as the solution of a boundary value problem with non-variational boundary conditions. Numerical results demonstrate the approximative solution of the optimal control problem by finite element discretization.

MSC: 49K20, 49M15, 65K10

Keywords: oblique boundary condition, non-variational boundary value problem, optimal control, optimality conditions.

1 Introduction

In this article we consider an optimal control problem for an elliptic partial differential equation with oblique boundary conditions. More precisely, we study the optimal control of the equation

$$-\partial_j(a_{ij}\,\partial_i y) + a_i\,\partial_i y + a_0\,y = f \quad \text{in } \Omega, \tag{1.1a}$$

$$b_i\,\partial_i y + b_0\,y = g \quad \text{on } \Gamma = \partial\Omega. \tag{1.1b}$$

*Accepted for publication in revised form on February 17, 2014

†Institut für Mathematik, Universität Würzburg, 97074 Würzburg, Germany, daniel.wachsmuth@mathematik.uni-wuerzburg.de

‡Technische Universität Chemnitz, Faculty of Mathematics, 09107 Chemnitz, Germany, gerd.wachsmuth@mathematik.tu-chemnitz.de