LINEAR DIFFERENTIAL GAMES WITH VECTOR-VALUED CRITERIA*

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Abstract

This paper deals with a problem of linear differential games with several quadratic objective criteria (with vector-objective). In this case the notion of Pareto min-max is used as optimum point of the differential game. We mention that the notion of Pareto min-max was introduced for the first time in [5]. Existence conditions (Theorem 1), necessary conditions (Theorem 2) and sufficient conditions (Theorem 3) are given.

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§ 1. Notations and Definitions

Let $\mathcal{X}$ and $\mathcal{Y}$ be real Banach spaces, $\emptyset \neq \mathcal{U} \subset \mathcal{X}$, $\emptyset \neq \mathcal{V} \subset \mathcal{Y}$ and $J : \mathcal{U} \times \mathcal{V} \to \mathbb{R}^m$, $m > 1$.

Definition 1. *Let $\mathcal{U}$ and $\mathcal{V}$ be convex sets. The function J is called convex with respect to $u \in \mathcal{U}$ and concave with respect to $v \in \mathcal{V}$ if and only if $J(\cdot, v) : \mathcal{U} \to \mathbb{R}^m$ is a convex function, $\forall v \in \mathcal{V}$ and $J(u, \cdot) : \mathcal{V} \to \mathbb{R}^m$ is a concave function, $\forall u \in \mathcal{U}$ (see [4]).*