

AROUND AN INEQUALITY, OR TWO, OF KY FAN*

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Abstract

In 1957 Ky Fan gave in [5] a necessary and sufficient condition, known as Fan's Consistency Condition, for a finite system of convex inequalities to have a solution. This result has been somewhat overshadowed by the famous Fan's Inequality which is equivalent to Brouwer's Fixed Point Theorem. Another result which bears Fan's name, but which is not due to him, is Fan's Lopsided Inequality which Aubin and Ekeland prove in [1] using Fan's Inequality.

We first prove a fairly general, but elementary result, Theorem 2.1.1, from which we derive both Fan's Theorem for finite systems of convex inequalities and Fan's Lopsided Inequality whose proof, therefore, does not require Brouwer's Fixed Theorem. We show that Theorem 2.1.1 is equivalent to Fan's Theorem for finite systems of convex inequalities; consequently, the Lopsided Inequality is a consequence of Fan's Theorem for finite systems of convex inequalities.

A number of well known and important results are proved along the way. The paths leading from Fan's 1957 theorem to those results are, we hope, simple enough to demonstrate that it deserves to be as well known as its younger and powerful cousin, Fan's Inequality.

MSC: 46A22, 46N10, 49J35, 49J40.

Keywords: convex inequalities, Fan's consistency condition. fixed points, Mazur-Orlicz.

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