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# MULTIGRID METHODS WITH CONSTRAINT LEVEL DECOMPOSITION FOR VARIATIONAL INEQUALITIES*

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**Abstract**

In this paper we introduce four multigrid algorithms for the constrained minimization of non-quadratic functionals. These algorithms are combinations of additive or multiplicative iterations on levels with additive or multiplicative ones over the levels. The convex set is decomposed as a sum of convex level subsets, and consequently, the algorithms have an optimal computing complexity. The methods are described as multigrid $V$-cycles, but the results hold for other iteration types, the $W$-cycle iterations, for instance. We estimate the global convergence rates of the proposed algorithms as functions of the number of levels, and compare them with the convergence rates of other existing multigrid methods. Even if the general convergence theory holds for convex sets which can be decomposed as a sum of convex level subsets, our algorithms are applied to the one-obstacle problems because, for these problems, we are able to construct optimal decompositions. But, in this case, the convergence rates of the methods introduced in this paper are better than those of the methods we know in the literature.

**AMS subject classification:** 65N55, 65N30, 65J15

**Keywords:** domain decomposition methods, variational inequalities, multigrid and multilevel methods

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