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# THE CONTROL VARIATIONAL METHOD FOR ELASTIC CONTACT PROBLEMS*

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**Abstract**

We consider a multivalued equation of the form $Ay + \partial\varphi(y) \ni f$ in a real Hilbert space, where $A$ is a linear operator and $\partial\varphi$ represents the (Clarke) subdifferential of the function $\varphi$. We prove existence and uniqueness results of the solution by using the control variational method. The main idea in this method is to minimize the energy functional associated to the nonlinear equation by arguments of optimal control theory. Then we consider a general mathematical model describing the contact between a linearly elastic body and an obstacle which leads to a variational formulation as above, for the displacement field. We apply the abstract existence and uniqueness results to prove the unique weak solvability of the corresponding contact problem. Finally, we present examples of contact and friction laws for which our results work.

**MSC**: 34B15, 34H99, 49K05

**keywords:** multivalued equation, Clarke subdifferential, control variational method, optimal state, elastic material, Signorini condition, normal compliance, Coulomb law of dry friction.

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