THE EFFICACY OF A LOSS SERVING SYSTEM

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Abstract: *Optimization solutions based on the application of expectation theory have a broad, but not exhaustive use, in the problems of analyzing and organizing networks in the composition of information systems. Modeling the effectiveness of a loss-serving system with Erlang's system seeks to reject the attack in terms of both economic efficiency and military efficiency.*

Keywords: *operational research, Erlang equations, mathematical modeling.*

I. Preliminary concepts

The theory of expectation is an essential component of operational research that studies the quantitative aspects of mass serving processes and highlights the organizational side of solving the process or phenomenon analyzed. It does not deal with the qualitative aspect of the serving units, considering them as a complementary, inherent side.

A problem based on the theory of expectation occurs in a process if both the intensity of requests arriving in the randomly assumed system and the number of service stations, as well as the related laws, can be studied and controlled.

Some examples of calculation based on theory of expectation on optimization in the military field can be mentioned[1]:

- Determining the optimum channel requirements for the communication networks (average number of channels occupied, average

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[1] Alexandrescu C, Ilina Decebal, Mincu C., *Bazele matematice ale organizării sistemelor de transmisiuni*, Editura Militară, Bucureşti, 1994, pp 20-60.

number of free channels, average probability of loss of communication flow, nominal transmission capacity of communications, etc.);

- Calculating the average value of the number of messages arriving at a command point, needed to define the flow of information capable of optimizing decisional problems;
- Calculating the number of simultaneous operation servers required to optimize the service of the workstations of the users on the computer network of a hierarchical echelon;
- Determining the likely performance of a firing system according to its technical tactical characteristics (number of batteries, duration of the firing cycle, average flow of the targets entering the fire area, etc.);
- Determining the likely outcomes of confronting adverse forces groups, the ratio of forces between them and the need for forces to optimize them;
- Calculating the need for materials to be provided in military action, by material categories and days of the operation.

A stand-by pattern is fully described by the following elements[2]: request flow, waiting string, serving stations, and outbound demand flow. Input flow analysis allows us to determine how requests arrive in the queue, considering that the incoming (arrivals) in the system are random and independent so that the likelihood that an application will come into the system is independent both when the arrival occurs and the number of applications previously existing in the system or the number of applications that will arrive.

Probability as in the time interval $(t, t+\Delta t)$, $t>0$, to produce an entry of a request into the system represents the average number of entries (arrivals) in the time unit Δt and is equal to $1/\lambda$ when it is considered that the arrival of requests is a Poisson process with the parameter λ, the flow of incoming requests into the serving system$(0 < \lambda < \infty)$.

Admitting that $t > 0$ and noting with $t_0, t_1, \ldots, t_n$ the successive moments in which the serving units arrive in the system, it is considered that the time intervals between two consecutive entries are identical random

[2] C. Alexandrescu, D. Ilina, C. Mincu, *Bazele matematice ale sistemelor de transmisiuni*, cap.5 Editura Militară, Bucureşti, 1994, pp.50-87.

variable distributions[3]. It is also accepted that serving times are random, independent and identically distributed random variables. From the point of view of how to serve requests, the mathematical models of serving systems can be[4]:

- loss-serving systems;
- waiting serving systems
- mixed serving systems (with limitation of number of serving requests in a row, with limitation of waiting time).

Loss-serving systems are those systems where a newly arrived request – if it finds all service channels occupied – leaves the system without being solved, and is therefore lost in serving, and the waiting time is zero. Wait-serving systems allow incoming requests to remain on hold, as the case may be, for an unlimited time, or requests to be served within a limited time (typically, a few minutes). Otherwise, they leave the serving system. Mixed serving systems admit certain limitations to loss-making systems in the sense of either limiting the length of the serving line or serving system requests.

II. Loss service systems

Most servicing systems characteristic of military action are loss-making, as they require immediate action (servicing) to ensure surprise capture on the enemy. They are especially specific to the operation of communications and computer networks, antiaircraft defense systems, enemy aviation systems, the fight against enemy tanks, etc.

To determine the number of service channels in a loss-making system, the following hypotheses should be started:

- the incoming calls flow (communication) is Poissonian;
- the Poissonian character of the input streams is maintained for both the (served) and the surplus traffic (not serviced);
- the service system is loss-making and is in a static statistical balance;

[3] Gh. Vrănceanu, Şt. Mititelu, *Probleme de cercetare operaţională*, Editura Tehnică, Bucureşti, 1998, pp.20-48.

[4] C. Alexandrescu, D. Ilina, C. Mincu, *Bazele matematice ale organizării sistemelor de transmisiuni*, Editura Militară, Bucureşti, 1994, pp. 107-111.

- no account is taken of traffic losses caused by switching and traffic management equipment in the network;
- service discipline is: first-come, first served FIFO (First In First Out).

Probability of service system states with loss in stabilized mode $p_k(t)$, respectively the probabilities $p_0, p_1,..., p_n$ as in the time interval t to enter requests (serving calls) made up of a simple stream that is calculated by the Poisson[5] formula, which can be expressed using the relation (1).

$$p_k(t) = \frac{(\lambda \cdot t)^k}{k!} \cdot e^{-\lambda \cdot t}, \quad 0 \le k \le n \qquad (1)$$

where:

k - number of service requests ($0 \le k \le n$) in the loss-making system;

n - total number of serving channels (stations) in the loss-making system;

$p_0, p_1,..., p_n$ - the probabilities of system states with stabilized losses;

λ - the density of the flow of requests for entry into the service system;

t - the amount of time required to serve a request.

Reality has shown that the flow of calls (communications) entered for all types of transmissions in the communications and computer networks is, as a whole, a simple stream, and the law of distribution of the time to serve the traffic. Since the flow of information is subject to Poisson's law, it will be admitted that the product *np* (where p is the probability value in the experiments) keeps a constant value, assuming that the average value of the number of occurrences of service calls in different series of experiments for different values of *n* remains unchanged. The average number of incoming communications (λ) over a time interval equal to the unit (e.g., hour) is expressed in relation to[6]:

$$\lambda = np \qquad (2)$$

[5] E. S. Venţel, *Teoria probabilităţilor*, Editura Nauka, Moskva, 1994, pp. 544-548.
[6] Pedersen, L. M., Nielsen, K. J., & Kines, P. (2012). Realistic evaluation as a new way to design and evaluate occupational safety interventions. Safety Science, 50(1), 2012, pp. 48-54.

The mathematical model for calculating the number of service channels (n) in a loss system is based on the Erlang[7] formula, defined in relation (3).

$$p_k = \frac{\frac{(\lambda_c \cdot T_m)^k}{k!}}{\sum_{k=0}^{n} \frac{(\lambda_c \cdot T_m)^k}{k!}} = \frac{\frac{y_c^k}{k!}}{\sum_{k=0}^{n} \frac{y_c^k}{k!}} \quad (0 \le k \le n) \tag{3}$$

in which:

t_m - the average time to serve a call (service requests) that ended with trafficking;

y_c - calculated traffic taking into account that in normal situations its value must be higher than that resulting from statistical data;

n - total number of channels existing in the loss-serving system;

λ_c - the flow density of the input requests;

Relationship (3) is true if the statistical balance (stationary regime) expressed in the $y_c<1$, defined by the y_c traffic and the number of n channels in the system is achieved in the service system analyzed. This balance is only achieved when the average number of incoming requests (calls) is lower than the number of channels. Also, the relationship (3) takes into account that the secondary stream of requests, consisting of those initially refused to serve but which subsequently occur, does not alter the Poissonian character of the input stream, and therefore the traffic in the networks, which happens when the probability of loss (non-maintenance of applications) is very low ($p_p \le 0{,}05$)[8].

In the Erlang formula (3), the size p_k represents the probability that at any time of random time for the stationary regime, from the total n channel for service on which flows a simple request flow (communication) with the λ parameter will be occupied *k* channel serving.

[7] C. Alexandrescu, D. Ilina, C. Mincu, Bazele matematice ale sistemelor de transmisiuni, Editura Militară, Bucureşti pp.109- 110.

[8] QFinance. (2010, June 22). Setting Up a Key Risk Indicator System. Retrieved from http://www.qfinance.com/operations-management-checklists/setting-up-a-key-riskindicator-system accessed October 10th, 2017

The value of the p_k probability of finding busy k channels depends only on the calculated y_c traffic and the total number of *n* channels and is not influenced by the service discipline, i.e. the order in which the free serving channels are occupied or the way the outgoing requests are set solved. Erlang's relationship is based on the assumption that the length of occupation of serving channels is distributed according to the exponential law.

If Erlang's formula calculates the limit case k=n, then the probability of loss p_p of missed calls in the serving system defined by relation (4) is obtained.

$$p_p = p_n = \frac{\frac{y_c^n}{n!}}{\sum_{k=0}^{n} \frac{y_c^k}{k!}} \tag{4}$$

In the particular case when the mathematical model of the loss-serving system has a single channel (k=n=1) the relation (5) is simplified taking the form:

$$P_p = p_1 = \frac{y_c}{1 + y_c} \tag{5}$$

With the formulas (4) and (5) you can calculate the probability of loss of traffic (demand line) when you know the incoming traffic y_c and the number of service channels available *n* or when the first two variables (p_p, y_c) implicitly obtain the required channel.

We have to point out that calculations are usually made for calculated y_c traffic, obtained by the formula:, respectively $y_c = t_m \cdot \left(\lambda + k_1\sqrt{\lambda}\right)$, where λ is the statistical value of the average demand flow in the calculated serving system, t_m is the time the environment for serving an application, and k_1 is a coefficient of importance established by the network designer (according to the normal distribution law, $k_1 = 0{,}6742$).

Problems in service systems are characterized by the fact that units entering the system when all its service channels are occupied can no longer be satisfied, are denied and leave the system, in other words, they are lost for servicing.

One kind of problem concerns the calculation of the effectiveness of some defense systems. We believe that there must be means ready to intervene immediately with fire to hit mobile targets at all times. The number of the latter depends on the number of discovered means and the possibilities of intervention, determined in turn by the technical means.

III. Example of calculation for the effectiveness of some defense systems

Let us consider a target attacked by enemy airplane with airplane frequency per minute, with anti-aircraft protection with *n=5* complexes for which the launch time is *0.5* minutes. In the case of the enemy attack with *N =20* planes, we propose to analyze the effectiveness of the antiaircraft defense system considering the probability of striking *P=0.7*.

We find ourselves in front of a loss-serving system. The initial condition, in which the flow of enemy airplanes is Poissonian, is acceptable because the requirement of leaving the formation (distances and intervals) is not rigorously observed, inherently occurring some random deviations, and under the action of the fire of anti-aircraft means the attacker's device is partial, the fire has a random character, but still maintains constant density. In addition, if the target is attacked from multiple directions, the total flow of attacking planes is even closer to a Poissonian flow. We add to the earlier that by accepting this hypothesis we find ourselves in the worst case that the defense has to solve, so the data obtained will be even more valid in other, simpler situations. The solving will not take into account the enemy's response.

The base data is $\lambda = 3$ airplanes / minute, $y_c = \lambda \cdot t_m = 3 \cdot 0.5 = 1.5$.

$$P_p = p_n = \frac{\frac{y_c^n}{n!}}{\sum_{k=0}^{n} \frac{y_c^k}{k!}}$$

$$P_5 = \frac{\frac{y_c^{\,5}}{5!}}{1 + \frac{y_c^{\,1}}{1!} + \frac{(y_c)^2}{2!} + \frac{(y_c)^3}{3!} + \frac{(y_c)^4}{4!} + \frac{(y_c)^5}{5!}}$$

$$P_5 = \frac{\frac{1,5^5}{5!}}{1 + \frac{1,5^1}{1!} + \frac{1,5^2}{2!} + \frac{1,5^3}{3!} + \frac{1,5^4}{4!} + \frac{1,5^5}{5!}}$$

$$p_5 = 0,031640625$$

The obtained data allow us to formulate the following conclusions on the effectiveness of the defense system:

- The probability p_p the system to be busy (of losses), as the planes are not subject to anti-air defense fire is $p_p = p_s = 0.031641$.

- The probability of fighting (immediate execution of the firing) of each target (airplane) involved in the attack is $p_f = 1 - p_s = 1 - 0,031641 = 0,968359$.

- The probability of destroying an aerial target is $p_d = P \cdot p_f = 0,7 \cdot 0,968359 = 0,6$.

-The average value of the number of planes fought will be $N \cdot p_f = 20 \cdot 0,968359 = 19,36718$.

- The average value of the number of planes destroyed, of those participating in the attack, is $p_d \cdot N = 0,6 \cdot 20 = 12$ planes

This also results in the number of planes reaching goal $N_0 = 20 - 12 = 8$. So, the duration of the attack will be T = N / λ = 20/366 minutes. In this time each complex will fire n = 20/6 = 3 targets using average for every 0.5 minutes.

I. Conclusions

In organizing a service system, it is necessary first of all to ensure the maximum satisfaction of requests in the serving system, and secondly, to serve with minimum consumption of forces and means and in the shortest possible time. Therefore, to solve this goal, the quantitative side of the serving processes and the registered efficacy should be studied.

Within the process, there is a discipline of service (expectation), so multiple sources with defined features can form multiple rows (waiting strings), and access to serving channels can be achieved based on a rule of priority.

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