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NUCLEAR STRUCTURE SNAPSHOTS. A PERSONAL SELECTION

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Abstract. *Several nuclear structure research directions are illustrated with results of different experiments from author's activity, and possible future developments are emphasized.*

Keywords: Nuclear structure, nuclear level lifetimes, nuclear deformation, high-spin states, direct transfer reactions

1. Introduction

The atomic nucleus is a complicated system of many nucleons (neutrons and protons) which interact through the so-called strong (nuclear) interactions. The experimental nuclear spectroscopy has as purpose to determine the characteristics of the different quantum states of this system: energy, spin, parity, lifetime, electromagnetic static moments (magnetic dipole, electric quadrupole), other quantum numbers, as well as the different decay modes of these states and their characteristics. The study of the electromagnetic (gamma) decay, which is the main decay mode of the excited states with energy below the particle emission threshold represents one of the most common means of deducing the properties of the excited nuclear states.

In order to study the excited states of a nucleus, they are populated by different nuclear reactions, or by decays. Each particular decay or reaction mechanism has specific features, therefore they will populate the final states preferentially; for example, the beta decay process will populate states in the daughter nucleus within a certain spin window, depending on the spin of the mother nucleus, whereas in fusion-evaporation reactions induced by heavy-ion beams one preferentially populates high spin states. It is therefore rather difficult to achieve a *complete* knowledge of the excited states, even at low excitation energies, in a certain nucleus, because one would have to combine many different methods of population and study.

There are a number of about 3000 nuclei for which spectroscopic measurements could be performed in different degrees of detail. Theoretical nuclear structure models predict the "existence" of other about 3000 nuclei, that is, nuclei within

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the (Z, N) limits defined by the nucleon (proton or neutron) drip lines, which live a long enough time before decaying -- thus giving the possibility to observe and study them with some experimental setups. This real "terra incognita" is far from the stable nuclei, and is rather difficult to reach. The near future will show a real revolution in the study of these exotic nuclear species, due to the commissioning of radioactive beam facilities (RIBs), such as FAIR (Darmstadt, Germany) and SPIRAL2 (Caen, France), and different dedicated experimental setups.

The motivation of the huge effort invested in these facilities is the new physics insight that will be gained. The prediction power of the present nuclear structure models is not too strong, because they were developed mainly based on the knowledge of the nuclei relatively close to stability. For example, physicists are looking forward to study new doubly magic nuclei, such as ^{100}Sn and ^{132}Sn, since until now only a few such nuclei could be reached; also, by going far from the nuclei studied in some detail until now, one expects that the shell structure will change, and thus the "classical" magic numbers will disappear and new ones will occur. It is also exciting to study the properties of new nuclei which are situated on the paths of different astrophysical nucleosynthesis processes (such as the *r*-, *s*-, and *rp*- processes), in order to be able to better understand how the present distribution of nuclei in the Universe was created. Therefore, nuclear spectroscopy will continue to be a rather dynamical field.

In this communication, several subjects of nuclear spectroscopy based on the author's results will be presented, and their actual interest and connection with future developments will be outlined.

2. High precision measurements of nuclear lifetimes with the DSA method

The lifetime of an excited state is a valuable experimental observable, since it determines directly the electromagnetic decay probabilities of that state towards other states. As shown in Fig. 1, the inverse of the lifetime τ (related to the half lifetime $T_{1/2} = \tau/\mathbf{ln}2$ is proportional to the so-called reduced electromagnetic transition probability from the initial (i) to the final (f) state $B(i \rightarrow f)$. The later is essentially the square of a matrix element between the wavefunctions of the two states, therefore it is sensitive to their structure.

The lifetimes of many nuclear excited states are in the range of about 0.01 ps to a few ps (10^{-12} sec), and one of the most suitable methods to measure them is based on the Doppler Shift Attenuation (DSA) method.

The principle is illustrated also in Fig. 1. The nuclei populated in the initial state i by some nuclear reaction in the target, will recoil (move) with a certain initial velocity v. Assuming that the recoils move in vacuum and we observe the gamma decay $i \rightarrow f$ at an angle θ with respect to the recoil direction, then the observed energy of the gamma ray will be Doppler shifted, according to the well-known formula (in first order) $E_\gamma = E_{\gamma 0}[1 + (v/c)\cos\theta]$. In this method we use, however, either a thick enough target, or a thin target with a thick enough backing, such that the recoiling nuclei are completely stopped. The slowing-down process, determined by the so-called stopping power (energy loss per thickness unit) has a certain mean time t_s. If τ is much smaller than t_s then practically all nuclei will decay after they were stopped, so that we will observe a "normal" peak, with the energy equal to $E_{\gamma 0}$. If, on the contrary, τ is much larger than t_s then all nuclei will practically decay when they still have the maximum velocity v, therefore we will observe a fully shifted peak with the energy E_γ defined above.

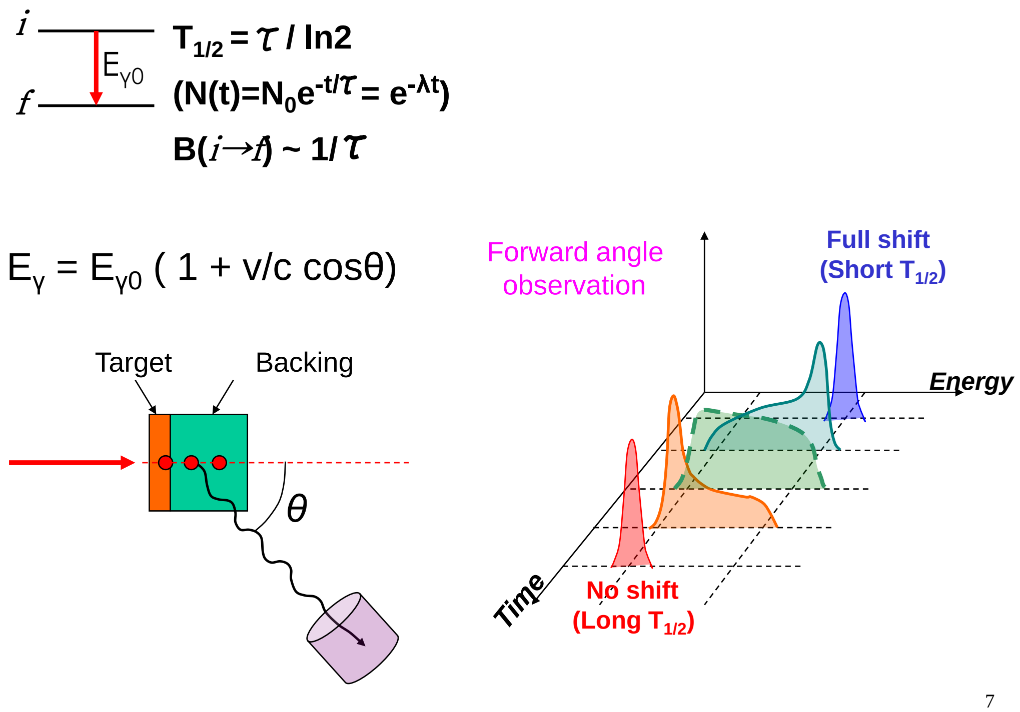

Fig. 1. Illustration of the DSA method for the lifetime measurements using the gamma-ray lineshape analysis.

For the case when the two times are comparable we will observe nuclei decaying during the whole stopping process, and the result will be a certain "lineshape", i.e. a distribution between the two extreme energies, with a shape which depends on the value of τ. We can observe in detail this lineshape if we have a smaller energy resolution compared to the maximum shift $(v/c)\cos\theta$.

The observation and analysis of a DSA lineshape is simple and efficient only if the recoiling nuclei have a well defined initial velocity, both in magnitude and direction, which is not the case in many reactions. There is, nevertheless, one case when such conditions are relatively easy to achieve, that of the inverse reactions, in which a heavy ion beam bombards a very light target. In this case the recoils are confined in a very narrow cone in the forward directions, and practically move with the velocity of the centre of mass.

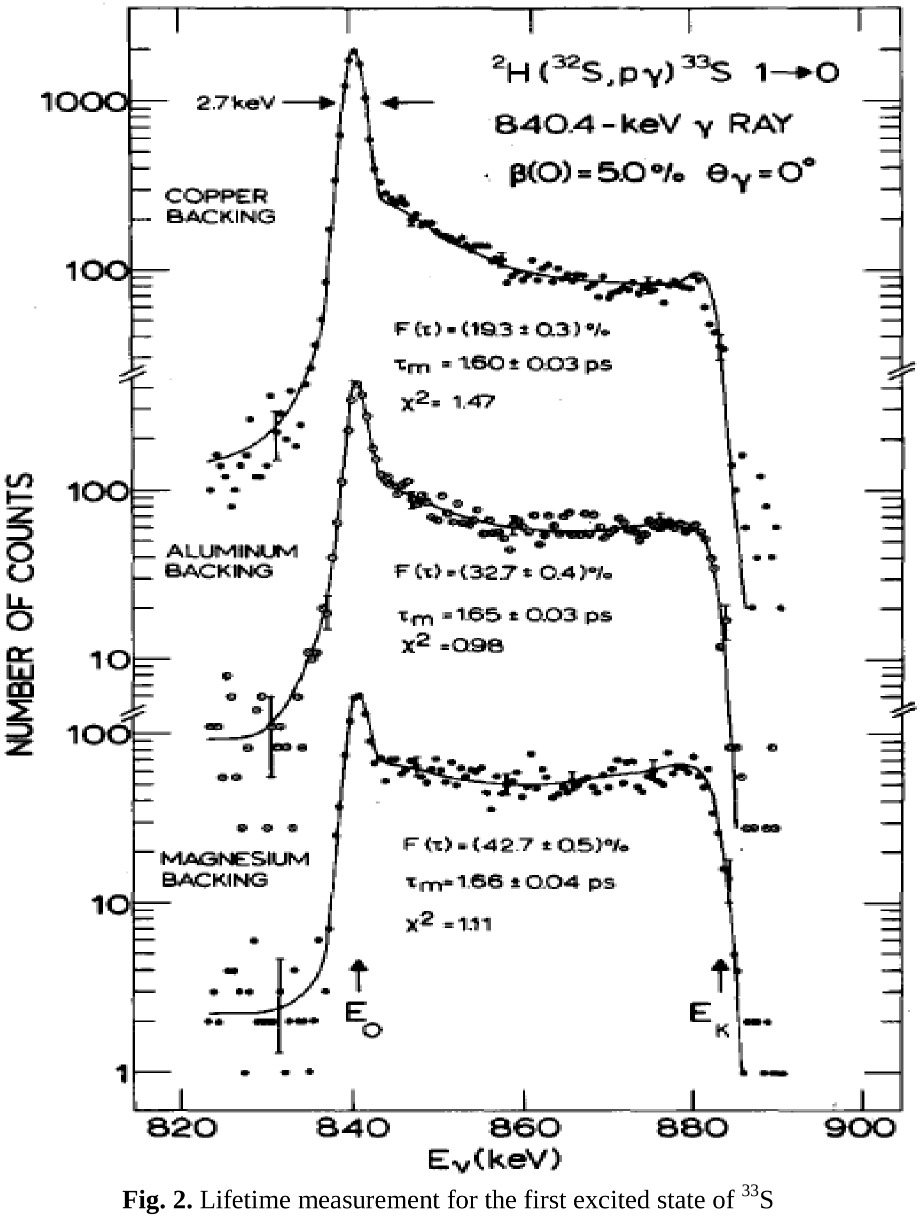

Fig. 2. Lifetime measurement for the first excited state of ^{33}S by the DSA method in the inverse (*d*, *p*) reaction [1, 2].

Fig. 2 illustrates one of the first measurements of this type [1, 2], in which the reaction used was $d(^{32}S, p)^{33}S$. The accuracy of the measurement was greatly increased by counting the γ-rays in coincidence with the protons detected in the backward direction with an annular counter, which further confined the direction of the recoils and, due to the possibility to select the excited state in the final nucleus ^{33}S, eliminated the effects of gamma-ray feeding from higher excited states. The DSA lineshapes of the gamma-ray transition from the first excited state in ^{33}S, for three different backings (copper, aluminium, and magnesium) and their analysis (Fig. 2) resulted in values of lifetimes consistent with each other within about 3% (their average is $\tau = 1.65\pm0.04$ ps), an excellent accuracy which is also due to the good knowledge of the stopping powers for such relatively high velocities of the recoils (in this case, $v/c = 0.05$).

The DSA method applied in inverse reactions will be a natural choice for many future experiments with exotic RIBs.

3. Shape transition in the nuclear regions A ≈ 80 and A ≈100

Nuclei are objects with a finite size, and may assume different shapes. Close to the magic numbers, nuclei have (nearly) spherical shapes, but those with numbers of neutrons (protons) in the middle of a major shell may be deformed, quite different from the spherical shape.

The most common type of deformation is the quadrupolar one, which is usually described by parameterizing the nuclear radius by

$$R(\theta,\Phi) = R_0[1+\beta \cos\gamma\, Y_{20}(\theta,\Phi) + (1/2)^{1/2}\, \beta \sin\gamma\, (Y_{20}(\theta,\Phi) + Y_{2,-2}(\theta,\Phi))],$$

where, for $\gamma = 0$ the nucleus is an axially symmetric elipsoid (called a *prolate,* or cigar shape), with β describing the ratio of the two axes, while for $\gamma \neq 0$ the nucleus becomes triaxial ($\gamma = 60^o$ describes an *oblate*, or pancake shape). Why nuclei deform can be explained by their microscopic, shell model description.

The nuclei have a shell structure, just like the atoms, but different in that it comes from a self-consistent finite range potential due to the strong interaction between all nucleons. This shell structure has certain groupings of the orbitals, separated by big energy gaps, which define the classical magic numbers: 8, 20, 28, 50, 82, etc. Most nuclei where the neutron and/or proton numbers are close to magic numbers are more stable due to the proximity of the big energy gaps. If we assume that the nucleons move in a deformed potential, which follows a shape as described above, one finds a new diagram of single particle levels (the so-called Nilsson diagram) which may present gaps at different other values of Z or N, but corresponding to a value of β different from zero. This means that nuclei may achieve a smaller total energy by taking a deformed shape rather than the spherical one.

Deformed nuclear shapes can be distinguished from spherical ones by looking at the energy spectra, electromagnetic decay probabilities, etc. of, e.g., the excited levels of the ground state band (g.s.b.) of the even-even nuclei, 0^+, 2^+, 4^+, ...

A spherical nucleus can excite by vibrations and will present a typical spectrum close to that of a harmonic vibrator: almost equidistant levels, E(*I*) = nhω, with n=*I*/2, *I* being the spin of the state. For a deformed nucleus, E(*I*) ~ *I(I+1)*. Also, the electromagnetic transition probabilities in deformed nuclei are much larger, since they imply the coherent contribution of many nucleons.

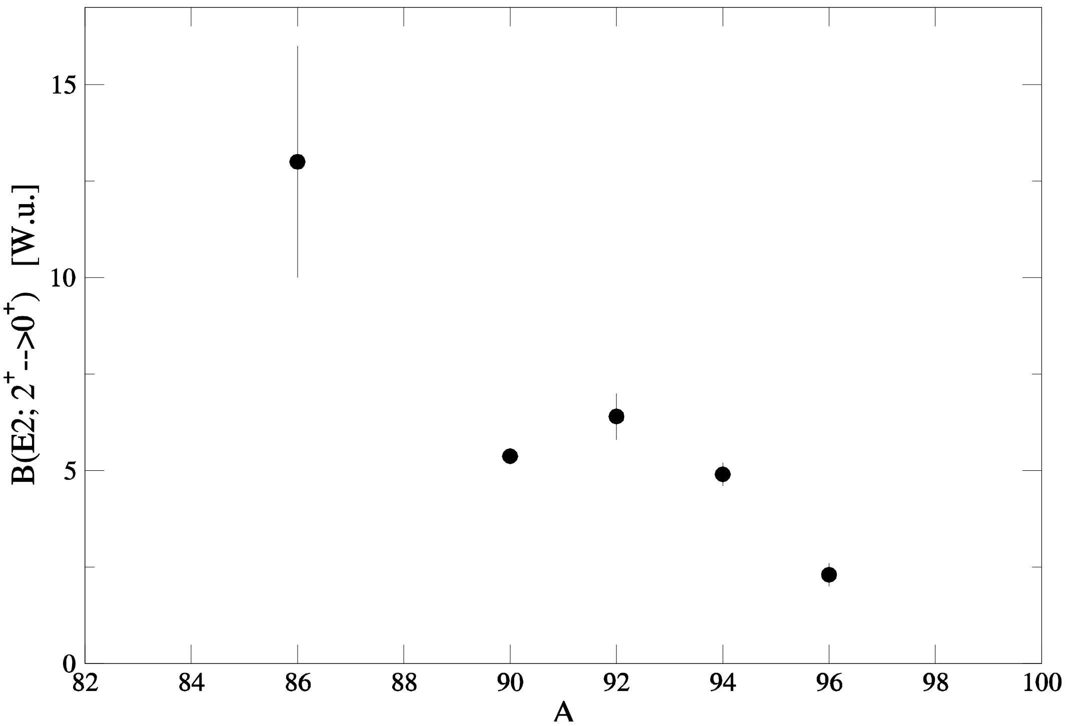

Fig. 3. B(E2) values for Zr isotopes 86 to 90;
only values available at the time of Ref. [3] are shown.

The first measurement of transition probabilities in neutron deficient Zr isotopes N < 48 was reported for ^{86}Zr, where the lifetimes of the levels in the g.s.b. were measured with the recoil-distance Doppler shift (or plunger) method in a heavy ion fusion-evaporation reaction, at the IFIN-HH tandem accelerator [3].

The result for the electric quadrupole transition probability B(E2; $2^+ \rightarrow 0^+$) is shown in Fig. 3, compared to those available for other, more neutron rich Zr isotopes. A B(E2) value increased by a factor of about 3 with respect to the value of the *N* = 50 - 56 isotopes clearly indicated that the lighter isotopes of Zr start to be deformed (in ^{86}Zr, the measured value determines a deformation β = 0.14. This was somewhat surprising, since it was thought that *Z*=40 might be a quasi-magic number for *N* < 50.

Although considered very interesting to measure lighter Zr isotopes, this was not possible at the time.

On the other hand, this interest led us to calculate how the nuclei in this region change when the number of neutrons is decreased. This was achieved by calculating potential energy surfaces (PES) with the macroscopic-microscopic method (where the nuclear potential energy is calculated as a sum of a macroscopic part, due to a deformed liquid drop, and the microscopic corrections due to the shell structure) [4]. The results for the neutron deficient Sr and Zr isotopes are shown in Fig. 4.

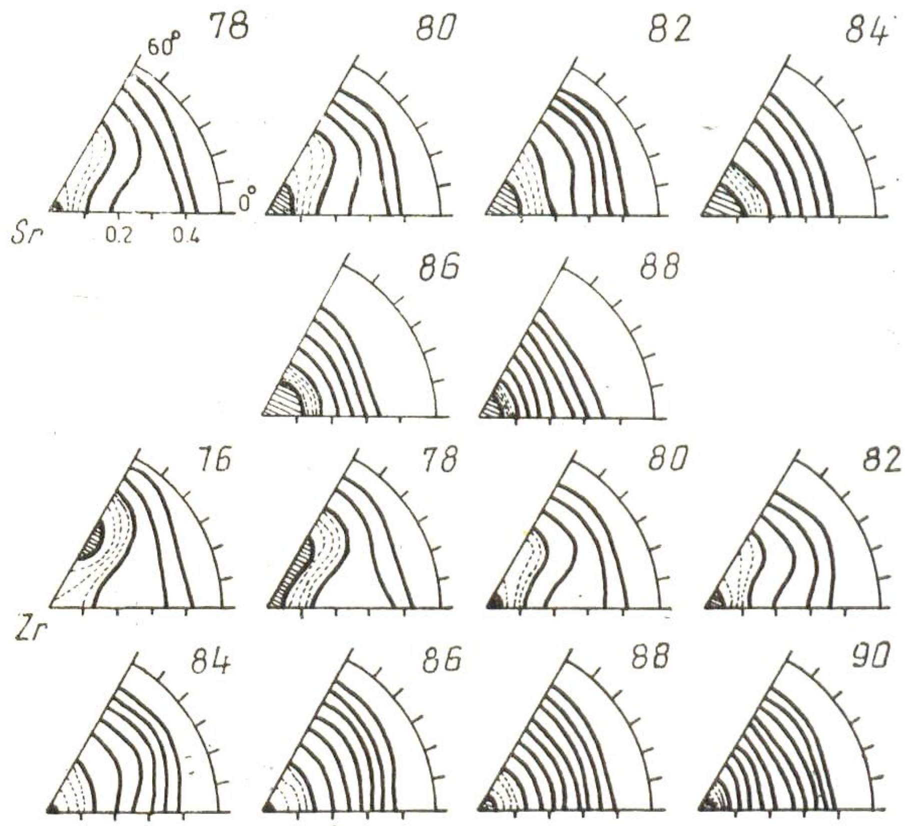

Fig. 4. Potential energy surfaces (PES) calculated for the neutron-deficient Sr and Zr isotopes. The iso-contours (in steps of 0.25 MeV) are drawn in the βγ plane (from Ref. [4]).

It can be seen that when N decreases toward the middle of the shell the nuclei have a clear tendency to achieve a permanent deformation, with the N=Z ones being the most deformed. Experimental studies which followed have found indeed that the Sr and Zr isotopes with N = 38,~40 are rather deformed [5]. New investigations [6] of these exotic isotopes, on or near the N=Z line, show that they are rather close to the so-called X(5) symmetry, which is the critical point of the shape phase transition between vibrators and deformed rotors [7].

The neutron-rich nuclei with A ≈ 100 is also a very interesting region, as a very sudden onset of large deformation was known to take place in the Sr and Zr isotopes when N passed from 58 to 60 [8]. Actually, in Ref. [6] this shape transition was rather well reproduced. The same type of calculations were performed for the Kr isotopes in Ref. [9], and they predicted that in this case there must be again a rather fast transition towards large deformation ($\beta \sim 0.30$) at $N = 60$ (^{96}Kr).

However, this heavy Kr isotope was not observed until now. Only recent investigations of our group with fission induced in the reaction ^{82}Se (500 MeV) + ^{238}U, in which gamma rays were registered with the CLARA array (25 Compton-suppressed clover detectors) in coincidence with the recoils identified with the PRISMA magnetic spectrometer [10], were able to assign the energy of the 2^+_1 state in this nucleus [11].

The systematic shown in Fig. 5 shows that the E(2^+_1) energy drops indeed very much between N 58$ and N=60 (see Fig. 5). According to current empirical relations between the E(2^+) value and the B(E2; 2+→0+) value, we expect indeed a deformation β close to the predicted value of 0.30.

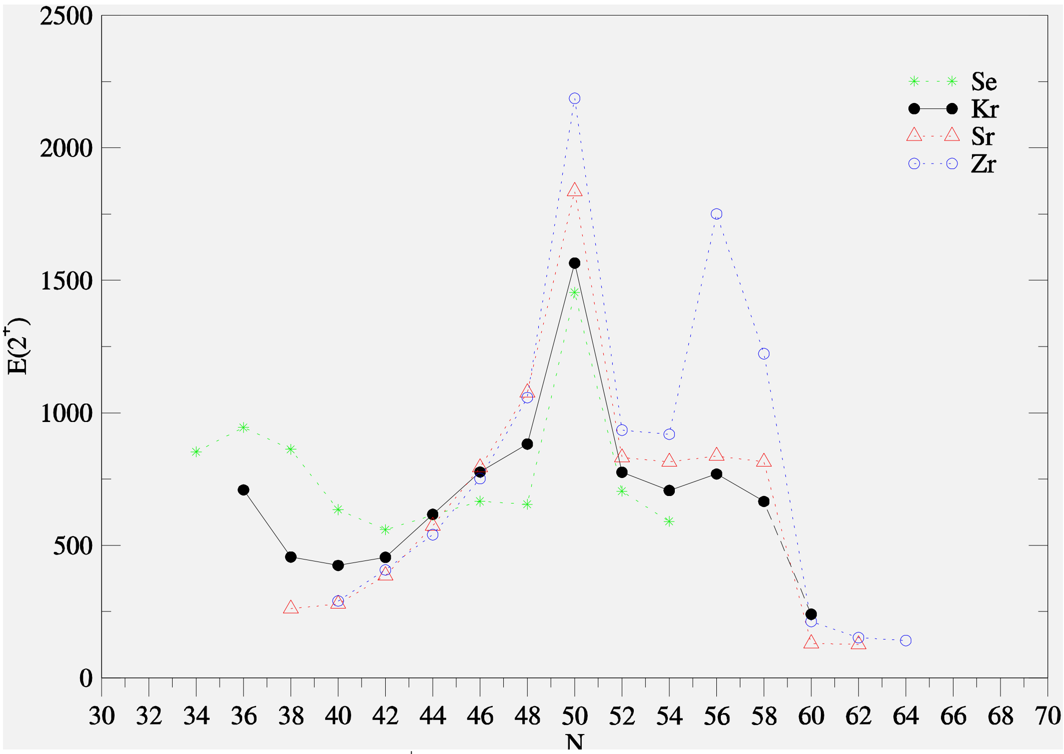

Fig. 5. Energy of the first excited 2^+ state in different isotopic chains, as function of the neutron number. One remarks the sudden drop of this quantity for ^{96}Kr, similar to the Sr and Zr isotopes [11].

A new experiment has therefore been proposed, to directly determine this deformation by measuring the B(E2) value of this transition by Coulomb excitation, at ISOLDE - CERN.

4. The heaviest N≈Z nuclei

As previously emphasized, the nuclear region with mass around 80 and $N \approx Z$ is rather interesting from the point of view of developing deformation. However, the study of the $N = Z$ nuclei is a subject of outstanding interest for the future nuclear spectroscopy studies, for a numberof other reasons, out of which we will remind only that of the *neutron-proton pairing*. As it is well known, the nuclei at low excitation energies are superfluid, because the neutrons and protons prefer to couple in pairs with anti-parallel spins ($S = 0$), due to the pairing residual interactions. This type of *pairing* between like particles is well established and studied. There may be also a neutron-proton pairing, but in this case we may have both pairs with $S = 0$, and with $S = 1$ (parallel spins, not forbidden by the Pauli principle because the two nucleons are not identical). The $S = 1$ *np*-pairing could not be studied until now, because the only nuclei where its effects might be visible are those with $N = Z$; as soon as N increases, the number of neutron-proton pairs is rapidly exceeded by that of proton-proton and neutron-neutron pairs, therefore the effects of the *np*-pairing are overwhelmed by those of the "usual" pairing.

One of the most conspicuous effects of the $S = 1$ *np*-pairing may be the following. In nuclei dominated by the usual *nn* and *pp* pairing, we normally observe that the g.s.b. presents a so-called backbending at a certain spin (usually around spin 8). This phenomenon can be explained as follows. Due to the rotational motion of the nucleus, the nucleons moving in the deformed field are influenced by the Coriolis force. The Coriolis force has opposite effects on the two nucleons of an S=0 pair (since they move in time-reversed orbits), and tends to break the pair. When the rotation (spin) of the nucleus increases, the Coriolis force increases too, and at a certain value it breaks the nucleon pair, the two nucleons aligning their spins: it becomes cheaper for the nucleus to build higher spin by aligning the spins of a pair rather than by increasing the rotation. This pair breaking is easily visible in the plot of the nuclear moment of inertia which shows a typical "backbending". However, in nuclei where the S=1 *np*-pairing is strong, the situation might be different: here the two nucleons move in similar orbits and therefore the Coriolis force does no longer destroy the pairs; one of the results of this lack of "Coriolis anti-pairing" effect is that in such nuclei we do not expect backbending as in the other nuclei.

The $N = Z$ nuclei in the A~80 region are notoriously difficult to reach. With the available (stable) beam-target combinations, they are populated with very small cross-sections, and therefore their observation with present instruments is very difficult. We performed systematic studies with heavy-ion fusion-evaporation reactions, using the GASP gamma-ray array in Legnaro\cite{GASP} coupled to different auxilliary detectors for neutrons and charged particles, and made spectroscopic studies of several $N = Z$ and $N = Z + 1$ nuclei which were observed for the first time. We show in Fig. 6 the results for the even-even $N = Z$ ^{84}Mo [13] and ^{88}Ru [14] nuclei (^{88}Ru is the heaviest $N = Z$ nucleus observed until now). More details can be found, e.g., in Ref. [15]. One observes that in practically all heavy $N = Z$ nuclei the expected backbending is missing, the evolution of the moment of inertia following a smooth curve up to the highest spin observed. This may be an indication on the existence of the *np* S=1 pairing. A more complete proof requires the investigation of the g.s.b. at higher spins than available now, and also the measurement of the lifetimes (transition probabilities) in this bands. Such measurements require much more efficient gamma-ray instruments.

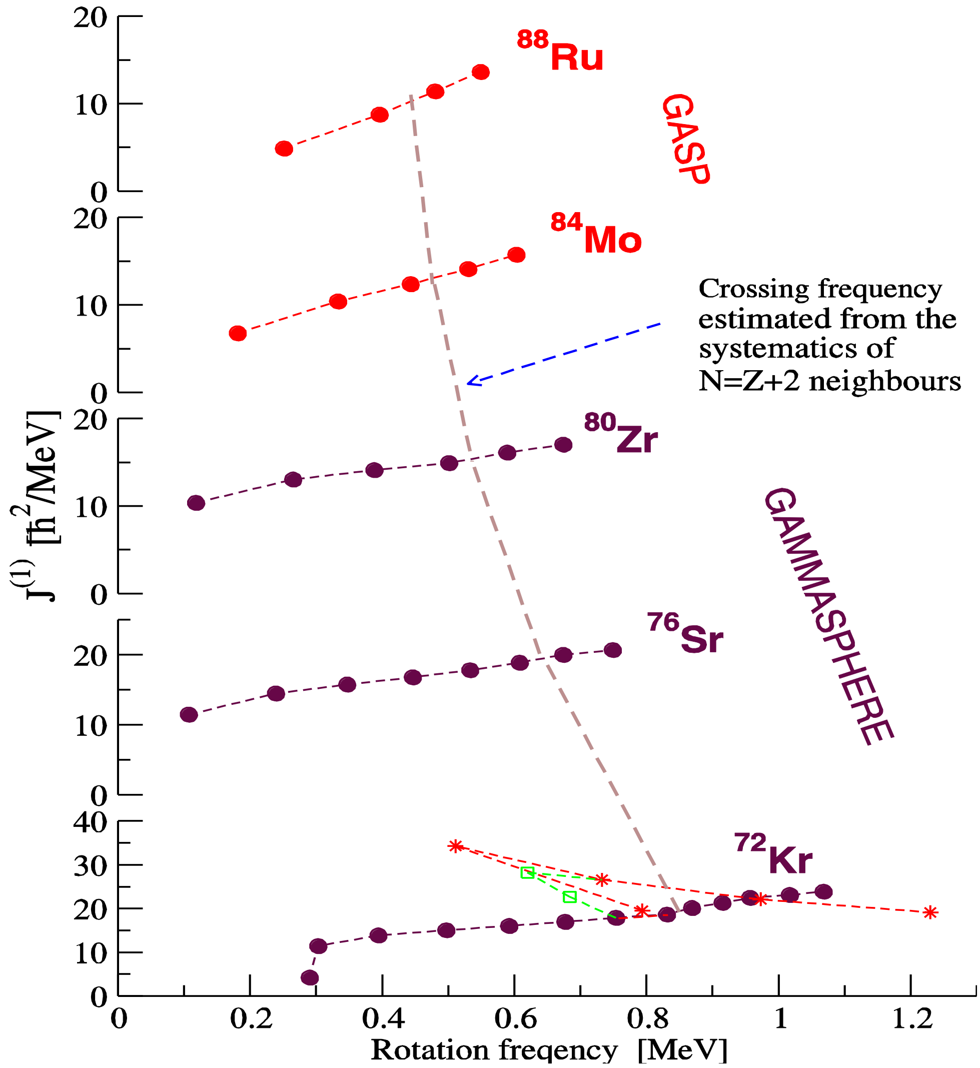

Fig. 6. Evolution of the moment of inertia in the heaviest known N=Z nuclei, as resulted from studies with the American array GAMMASPHERE and with GASP, as quoted in the text. The dashed line indicates the place where, according to systematics along the isotopic chains, one should observe backbending (from Ref. [15]).

5. Quasicomplete spectroscopy with direct transfer reactions

As remarked in the introduction, different nuclear reactions used to populate states in a nucleus under study have different selectivities concerning the number and type of excited states. In this chapter we present a different type of approach, which is somewhat complementary to the methods of gamma-ray spectroscopy: the spectroscopy with direct transfer nuclear reactions induced by light particles. This type of particle spectroscopy studies was rather usual in the seventies, however with energy resolutions much weaker than that usual with the gamma-ray spectroscopy.

Nuclear reactions of the direct type (to fix ideas we note a "typical" such reaction, the (d, p) stripping reaction) excite preferentially states in a certain spin window (usually low spin states) in which the final state is made out of the initial (ground) state by a minimum re-arrangement of the nucleons in the target nucleus: for example, in the (d, p) reaction, the states populated with sufficient strength to be observed, are those which have a relatively strong component in the wavefunction in which the added neutron occupies one of the empty orbitals of the target nucleus. Thus, direct reactions of different types test the occupancy/emptiness of the shell model orbitals, and the nucleon-nucleon correlations. Such reactions bring complementary information to fusion evaporation reactions which populate mostly collective and high-spin states.

QPM PSM 6^{th} order boson H

We give here as an example such a direct reaction study of the ^{168}Er nucleus by the ^{170}Er(p,t) reaction at 25 Mev incident energy. This study (Ref. [16]) is outstanding in that it was performed with a very good energy resolution, about 5 keV FWHM, which allowed the identification of a large number of excited states (around 150) up to an excitation energy of 4.1 MeV, where the density of levels becomes rather high. This exceptional resolution was achieved by using the Q3D magnetic spectrograph of the Technical University and LM University in München [17], with a 1 meter long focal plane detector [18].

In Ref. [16] we concentrated on the observation of the 0^+ and 2^+ states, which can be easily recognized by their typical angular distributions. An unusually large number of such states was observed: 26 0^+ states and 64 2^+ states, most of them new, in ^{168}Er, which was considered one of the best studied nuclei.

This very large number of states makes possible two things:

(i) the extension of the comparison with predictions of state-of-the-art nuclear models in the region were the level density becomes high;

(ii) study of the order/chaos properties of the nuclear structure on pure ensembles of levels (in this case, with just one spin value).

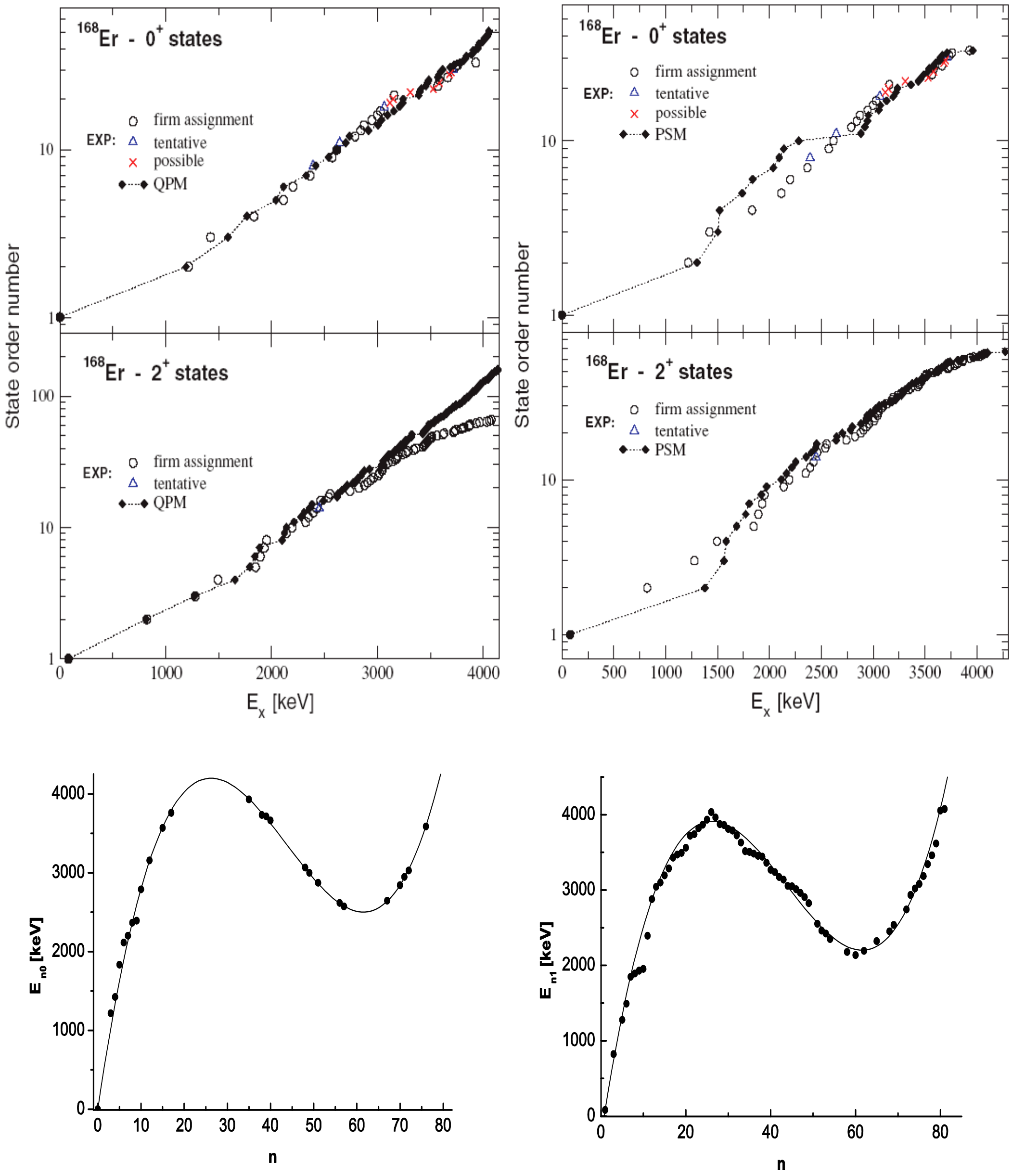

Fig. 7. Comparison of observed 0^+ and 2^+ states in ^{168}Er with calculations (*see text*).

In Fig. 7 we show a comparison of the observed 0^+ and 2^+ levels with predictions of three models: the quasi-particle phonon model [19], the projected shell model [20], and a model based on a 6th order boson hamiltonian [21]. It is seen that the QPM model describes rather satisfactorily the observed levels up to about 3.2 MeV excitation (above that it is probable that we did not observe all the existing 2^+ levels), while the PSM model in its actual variant has some problems.

The use of the model of Ref. [21] raises interesting questions regarding the classification of the large number of observed states in terms of underlying number of bosons.

6. Gamma-ray spectroscopy at the IFIN-HH tandem accelerator

Here it is briefly mentioned a continuous activity of gamma-ray spectroscopy in fusion-evaporation reactions with heavy ions, which was based initially (as mentioned in Ref. [3]) on very modest experimental setups (one or two small Ge(Li) detectors). At present we have a small array of seven HPGe with efficiencies around 55% (total absolute efficiency of about 0.7% at 1.33 MeV), as shown in Fig. 8, where an additional number of 5 $LaBr_3$ detectors can be seen. Also, there are two clover HPGe detectors, and a new plunger device with piezoelectric control of the target to stopper distance was built. We illustrate this type of research with recent results concerning the odd-odd nucleus ^{86}Y. Several states were assigned to the yrast positive parity line in this nucleus through the (^{14}N,4n) and (^{16}O,p2n) reactions [22]. Recently, we have re-measured this nucleus both with the (^{14}N,4n) reaction in Bucharest, and the ^{52}Cr(^{37}Cl, 2pn) reaction with GASP in Legnaro (which allowed the observation of higher spins).

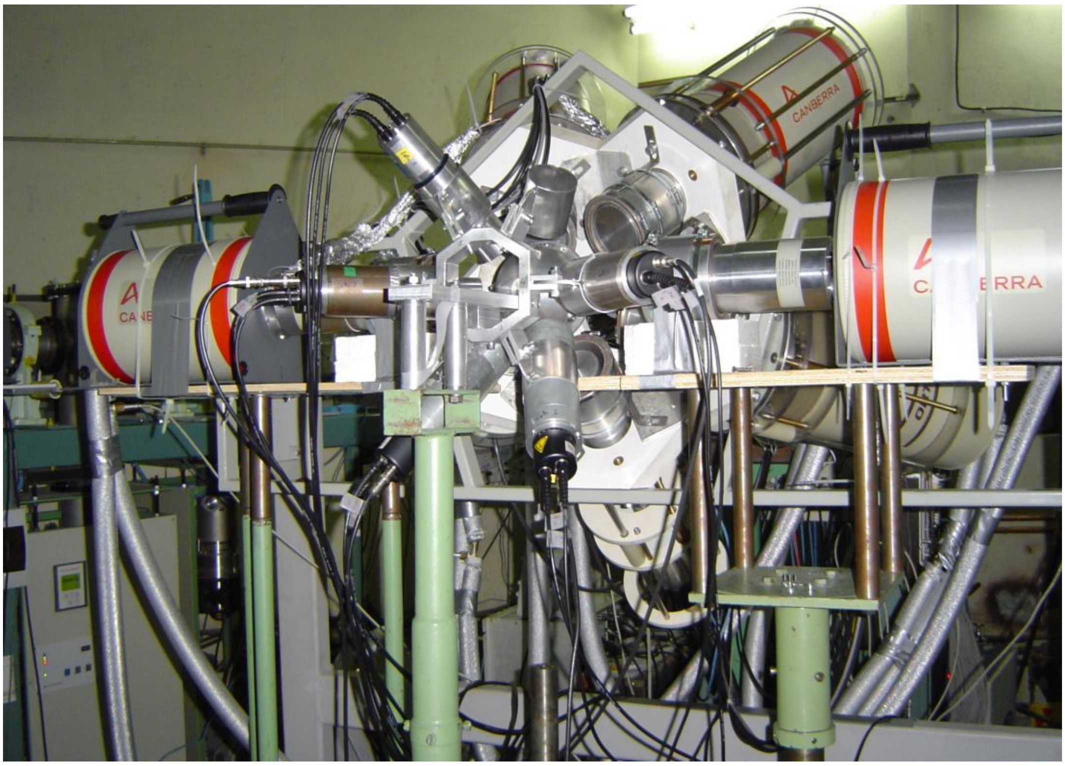

Fig. 8. Gamma-ray spectroscopy setup at the Tandem accelerator laboratory of IFIN-HH.

The resulting level scheme, published very recently [23] is shown in Fig. 9.

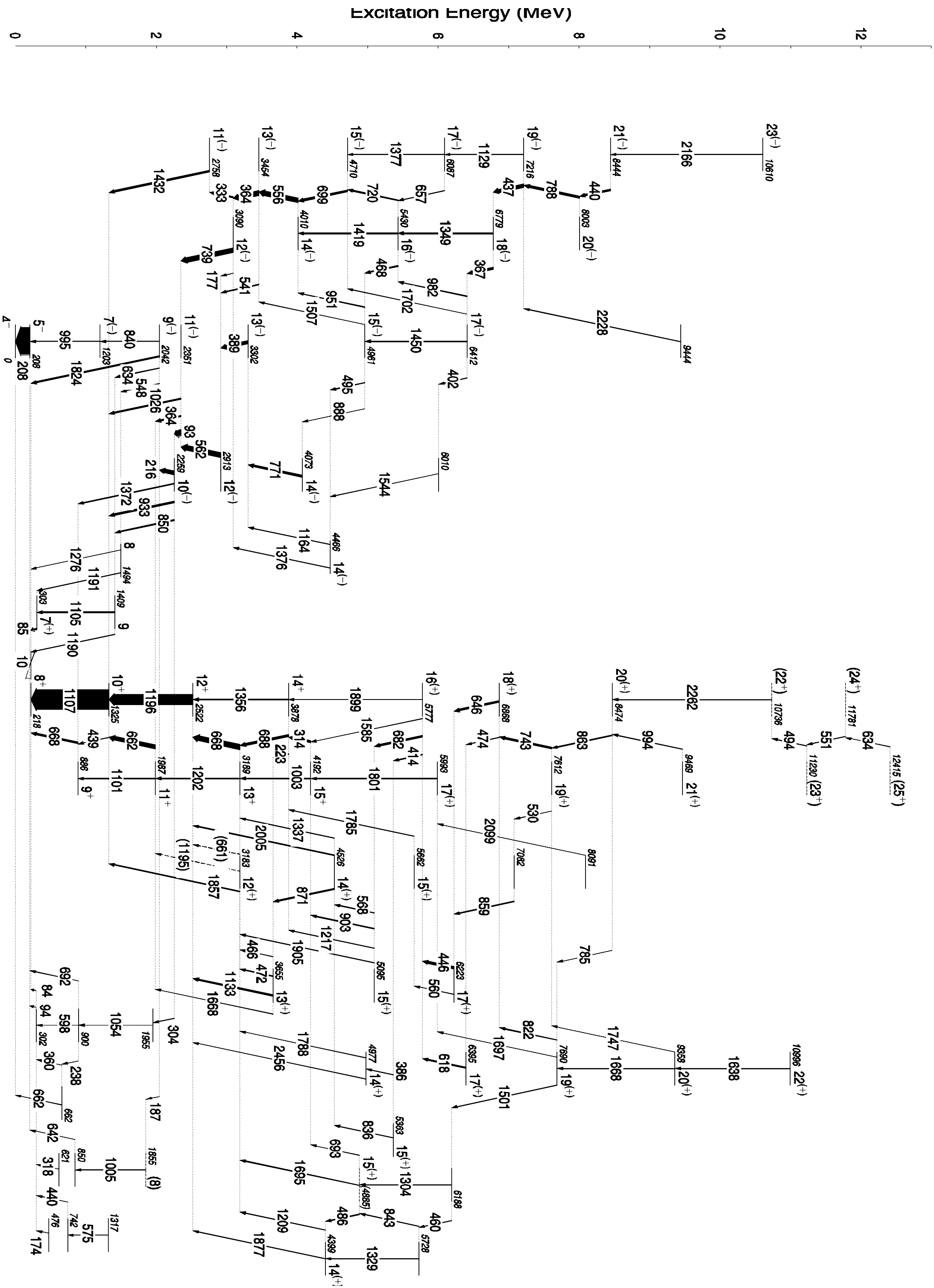

Fig. 9. Level scheme of 86Y as observed in recent heavy-ion fusion-evaporation studies [23].

Notable in this level scheme are the sequence of negative parity levels joined by a quasi-regular M1 transition cascade (left upper part of the figure), which is similar with "magnetic rotation" bands observed in Rb isotopes, and the continuation of the positive parity sequence up to 12 MeV excitation and spin 25, which clearly shows the occurrence of ^{88}Sr core breaking (the large energy transitions around spin 16) and, at higher spins, of two particle-two hole excitations.

This rich level scheme was very well explained by shell model calculations in which up to two nucleons (neutrons or protons) were allowed to be excited from the $f_{5/2}$ and $p_{3/2}$ shells in the upper $p_{1/2}$ and $g_{9/2}$ shells [23].

New gamma-ray spectroscopy measurements with the setup in Fig. 8 and different types of reactions, devoted to level scheme construction and lifetime measurements with different methods (DSAM, plunger, and fast electronic timing) are under way.

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