SKEW PROJECTORS AND GENERALIZED OBSERVABLES IN POLARIZATION OPTICS

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Abstract. The non-Hermitian operators of the non-orthogonal multilayer optical polarizers represent observables in the sense of the generalized quantum theory of measurement. The intimate spectral structure of these polarizers can be disclosed in the frame of skew-angular vector bases and bi-orthonormal vector systems. We show that each of these polarizers corresponds to a skew projector, its operator is "generated" by a skew projector, in the sense of the spectral theorem of linear operators theory. Thus the common feature of all the polarizers (Hermitian and non-Hermitian) is that their "nuclei" are (orthogonal or skew) projectors — the generating projectors.

Keywords: non-Hermitian operators, non-orthogonal multilayer optical polarizers, skew-angular vector bases

1. Introduction

In the last decades some extensions of the standard Dirac – von Neumann measurement formalism in quantum mechanics were elaborated [1-5].

In the standard formalism, a measurement corresponds to a Hermitian operator yielding its eigenvalues as measurement results, with probabilities determined by the values of the orthogonal projection of the system state on the operator's eigenvectors. In other words an observable is a "projection-valued measure" (PVM).

In a fundamental paper by E. B. Davies and J. T. Lewis [6], the concept of generalized observable is described, which arise when two standard non-commuting observables \boldsymbol{a} and \boldsymbol{B} are measured one after the other. The class of observables is extended to the positive operator-valued measure (POVM). Particularly the POVM becomes a PVM when the two standard observables \boldsymbol{a} and \boldsymbol{B} commute.

In the standard theory of quantum measurement the postulate of the repeatability plays a central role: if a physical quantity is measured twice in succession in a system, one gets the same value each time. This hypothesis is equivalent to the fact that the class of observables is restricted to (orthogonal) projectors. In the generalized theory of quantum measurement the postulate of repeatability is abandoned.

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